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A COMPARISON OF MATHEMATICS ACHIEVEMENT AFTER DISCOVERY AND EXPOSITORY TEACHING IN GRADE ELEVEN

by

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A THESIS
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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "A Comparison of Mathematics Achievement After Discovery and Expository Teaching in Grade Eleven," submitted by Gerald M. Tobert, in partial fulfilment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of this study was the comparison of student mathematics achievement as a result of exposure to a grade eleven unit on quadratic functions and equations taught by two different teaching methods.

In January, 1968, seven teachers, each of whom taught two classes of Mathematics 20 in the Edmonton Public School System, attended in-service training sessions designed to acquaint them with the Mathematizing Mode (M), a defined discovery method, and the Expository Mode (E), a defined expository method. For approximately a seven-week period these teachers taught one class using the M mode (making up the M group) and the other using the E mode (making up the E group).

The writer prepared three examinations of student achievement based on Bloom's Taxonomy; a pre-treatment test, a post-test, and a retention test. These examinations were administered to all students in the project before, immediately after, and seven weeks after the experimental period, giving the necessary data.

The prime consideration was to test for mean differences between the M and E groups on the instruments prepared. The groups were each divided into three IQ levels containing equal numbers of students; Lo, Ave, and Hi. Thus it was possible to assess the effects of the two different treatments on students of three different ability levels. Two-way analysis of variance was carried out on the data to test the null hypotheses.

On the basis of the analysis, the following results, within the limitations of this study, were obtained.

There was no significant difference between the M and E groups on a pre-test of student achievement.

There was no significant interaction between modes of instruction and IQ levels on the post-test and retention test.

There was a significant difference between the M and E groups on a post-test of student achievement, favoring the E group.

There was a significant difference between the M and E groups on a retention test of student achievement, favoring the E group.

This study would indicate a superiority for the defined Expository method over the defined Mathematizing method with respect to promoting student achievement for the material covered during the experiment.

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CHAPTER I

THE PROBLEM

Introduction

Education is a dynamic, growing and changing field. The objectives of instruction in almost every discipline, especially that of mathematics, are under constant close scrutiny to determine the proper emphasis of these objectives to meet the demands of our changing world. The methods of mathematics instruction are also changing in hopes of achieving a suitable emphasis of these objectives. There is a spirit of experimentation among mathematics teachers, demonstrated by the fact that many are willing to try any new instructional technique that holds promise of meeting new emphases. Most progressive mathematics educators now realize that to be effective in meeting the shifting emphases on objectives of instruction, they must be proficient with many instructional techniques and know for what purpose to use each of these.

Background to the Problem

Theoretical Aspects

Traditionally, mathematics has been taught in an expository manner; that is to say, by use of a teacher-centered lecture or tell-to-do method. Currently, there is a great deal of interest, on the part of mathematics educators, in the discovery methods of teaching. By this is usually meant a method whereby "the learner completes

a learning task without extensive help from the teacher." (Bittinger, 1968, p. 140)

On the theoretical level, little agreement is found among scholars as to the value of discovery teaching. Proponents of discovery tend to make extensive claims for this approach; for example, Bruner says, "The most uniquely personal of all that he (anyone) knows is that which he has discovered for himself." (Bittinger, 1968, p. 141) Bruner also insists on the following advantages of discovery:

> (1) increase in intellectual potency, (2) intrinsic reward, (3) learning the heuristics of discovery, (4) conservation of memory. Increase in intellectual potency is the ability to assemble material sensibly. Learning by discovery provides intrinsic pleasure in finding out for one's self. Learning the heuristics of discovery, according to Bruner, is the learning of attitudes and activities that go with inquiry and research and is only achieved through the act of problem solving and the effort of discovery. . . . Discovering things for one's self also seems to have the effect of making material more readily accessible to memory. (Bittinger, 1968, p. 141)

Suchman, another proponent of discovery, feels that

> Because learning through inquiry (discovery) permits greater self-direction and self-evaluation, the student is more motivated and freer to pursue knowledge and understanding in accordance with his cognitive needs and his individual level and rate of assimilation. Also, concepts will have greater significance to the child. (Bittinger, 1968, p. 141)

Those who have reservations about the use of the discovery method do exist. For instance, Ausubel feels that "most of what anyone really knows consists of insights discovered by others which have been communicated to him in meaningful fashion." (Bittinger, 1968, p. 141) Further, Ausubel is of the opinion that

> On developmental grounds this technique is generally unnecessary and inappropriate for teaching subject-matter content, except where pupils are in the concrete stage of cognitive

> development. During the concrete stage, roughly covering the elementary school period, children are restricted by their dependence on concrete-empirical experience to a semi-abstract, intuitive understanding of abstract propositions. (Ausubel, 1964, p. 292)

In other words, Ausubel feels that discovery teaching is most valuable in the elementary school. Once students reach high school age, they are in the abstract stage of cognitive development, and discovery learning may be of little benefit. To again quote Ausubel:

> During the abstract stage of cognitive development, however, the psychological rationale for using discovery methods to teach subject-matter content is highly questionable. Students now form most new concepts and learn most new propositions by directly grasping higher-order relationships between abstractions. To do so meaningfully, they need no longer depend on current or recently prior concrete-empirical experience, and hence are able to by-pass completely the intuitive type of understanding reflective of such dependence. Through proper expository teaching they can proceed directly to a level of abstract understanding that is qualitatively superior to the intuitive level in terms of generality, clarity, precision and explicitness. At this stage of development, therefore, it seems pointless to enhance intuitive understanding by using discovery techniques. (Ausubel, 1964, p. 293)

It appears then that just when and how much the discovery approach should be used, as well as its over-all effects on the student, remain open questions. In short, the place of discovery teaching in mathematics education has yet to be determined.

Research Aspects

A great many outcomes of both discovery and expository teaching have been examined by researchers. This has been done through many comparison studies of the "discovery versus expository" type, where comparable populations of students are taught using these two methods, and many tests are administered to the respective populations in an

attempt to determine differences in many aspects of mathematics learning, including the vital aspect of student achievement. Thus far, no definite conclusion has been reached as to the relative merits of discovery and expository teaching with regard to student achievement. Kersh, in discussing this, states:

> There is research evidence to support the claim that learners become more interested in whatever they are learning, but the evidence is equivocal with respect to the claim for increased understanding, and practically non-existent with respect to the (third) claim concerning the attainment of complex learning processes. (Kersh, 1964, p. 226)

Even where evidence favoring the discovery approach exists, Ausubel and Bittinger point to a serious problem which frequently plagues such research: rarely does one find the expository group treated as an experimental group. Ausubel insists that researchers, in many instances,

> . . . have not even attempted to obtain comparable achievement test data from matched control groups. And only rarely has any effort been expended to prevent the operation of the crucial 'Hawthorne Effect', that is, to make sure that evidence of superior achievement outcomes is attributable to the influence of new pedagogic techniques or materials in question, rather than to the fact that the experimental group is the recipient of some form of conspicuous special attention. (Ausubel, 1964, p. 300)

One other point worthy of note concerning research on discovery teaching in mathematics, is that practically no studies of the discovery versus expository type have been conducted at the high school level. Perhaps this is unconscious support for Ausubel's claim cited earlier, that discovery becomes progressively less useful as a teaching technique with the advancing age of the student, and is of little value to the high school pupil.

Statement of the Problem

The prime purpose of this study was the comparison of student achievement in mathematics as a result of discovery and expository teaching. A definite attempt was made to meet the reservations about studies of this nature in that this study was conducted as part of a large project at the senior high school level and made use of a truly experimental expository group.

Context of the Problem

This study was part of a large, complete group project which compared discovery teaching with expository teaching in high school mathematics on several dimensions. Five graduate mathematics education students, one of whom was the writer, assumed responsibilities for investigation of various dimensions. One student was responsible for the investigation into the nature of discovery teaching and the production of suitable discovery teaching materials. Another was responsible for the area of teacher behaviors in a discovery class as opposed to an expository class. A third graduate student investigated pupil creativity as a result of discovery and expository teaching. A fourth dealt with student attitudes and reactions to both teaching methods. The fifth, the writer, was responsible for measuring student achievement before, immediately after, and a period of time after the two different treatments.

Nine teachers, each of whom taught at least two classes of eleventh grade Mathematics 20 in the Edmonton Public School System, attended a five-session, in-service program designed to acquaint them with the Mathematizing Mode, a discovery method, and the Expository

Mode, an expository technique. The intent of the in-service program, conducted by two university Mathematics Education professors, was to enable teachers to develop the patterns of behavior which would permit them to effectively teach one class using the Mathematizing Mode and the other class using the Expository Mode. Each teacher was thus required to teach two units on the quadratic function of Mathematics 20 to one class using the Mathematizing Mode and to the other class using the Expository Mode for a period of appoximately seven weeks. The two methods used were characterized as follows:

Mathematizing Mode:

1. An initial exploration of a problem area by students essentially unguided by the teacher.
2. Formulation of hypotheses by students.
3. Testing of hypotheses by students (some teacher guidance).
4. Proper generalization of correct hypotheses into standard rules or laws (some more teacher guidance).
5. Students given some practice in applying rules or conclusions.[1]

Expository Mode:

1. Strong teacher control of the learning situation. The teacher acts as the agent with the answers.
2. The teacher exposes; the student looks, thinks, acts; the teacher evaluates.
3. The presentation in class is basically a lecture or

---

[1] For an elaborate description of the Mathematizing Mode, see Johnston, 1968, pp. 41 - 62.

structured discussion accompanied by visual materials and easily-noted important rules. The teacher states the final conclusions.

4. Students are given plenty of practice in applying rules or conclusions.[2]

Since by nature a discovery approach is more time-consuming than an expository approach, it seems likely that more opportunity for practicing the application of rules or laws developed was available under the Expository Mode.

Hypotheses

The principal hypotheses tested were:

1. There is no significant interaction between modes of instruction and level of intelligence on an investigator-made, post-experiment examination of content learned by students taught in the Mathematizing Mode and the Expository Mode.
2. For Mathematics 20 students of high, average and low mental ability, there is no significant difference in achievement on an investigator-made, post-experiment examination of content learned between students taught in the Mathematizing Mode and those taught in the Expository Mode.
3. There is no significant interaction between modes of

---

[2] For a more elaborate description of the Expository Mode, see Appendix A.

instruction and level of intelligence on an investigator-made examination of retention (after seven weeks) of content learned by students taught in the Mathematizing Mode and the Expository Mode.

4. For Mathematics 20 students of high, average and low mental ability, there is no significant difference in achievement on an investigator-made examination of retention (after seven weeks) of content learned between students taught in the Mathematizing Mode and those taught in the Expository Mode.

Delimitations of the Problem

The investigation was limited by the extent to which teachers participating actually consistently taught their classes in the Mathematizing Mode and Expository Mode.[3] The results of the investigation undoubtedly hinge on the reliability of the instruments prepared to measure the achievement of the students.[4] Also, this study was limited to a sample of the total population of Mathematics 20 students involved.[5] The time available for the study and relative unfamiliarity of students and teachers with the Mathematizing Mode are further limitations on the study.

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[3] This aspect was dealt with by another investigator. See Appendix B for relevant material.

[4] See Chapter III, p. 41 of the present study.

[5] See Chapter III, p. 36

Significance of the Problem

As previously stated, existing discovery versus expository studies on record have produced no conclusive evidence of the superiority of either method in regard to student achievement. In addition, there is the theory of Ausubel which states, in effect, that discovery teaching is of little value at the high school level. Also, one must note that no studies of this nature have been done at the high school level in Canada. And finally, previous studies of this nature have failed to make the expository treatment a proper experimental method.

The main significance of the present study is that it provides objective evidence of the effectiveness of discovery teaching in mathematics on student achievement at the high school level. It also makes experimental treatments of both the discovery and expository modes, which should permit more effective comparison of them. As a result, the study should help to determine the value and place of discovery teaching in mathematics education.

Experimental Procedure

The investigation reported here is one part of a group project. Nine teachers, each of whom taught at least two classes of eleventh grade Mathematics 20 in the Edmonton Public School System, were involved. Since each of the teachers was required to teach at least two classes of Mathematics 20 for the purposes of this study, data was obtained only from students of seven teachers.

Each teacher attended five two-hour, in-service sessions in January and February, 1968. The in-service sessions, conducted by

two University of Alberta Mathematics Education professors, consisted of lectures and discussions on the Mathematizing and Expository methods of teaching quadratic functions of the Mathematics 20 course. Participating teachers were given materials explaining the activities to be carried out in teaching quadratic functions using the Mathematizing Mode and the Expository Mode.

During the period February to May, 1968, each teacher was required to teach one class using the Mathematizing Mode, while the other class was to be taught using the Expository Mode. Classes were assigned randomly to the methods.

Intelligence quotients of pupils involved in the study were obtained from existing school records. Data on pre-experimental student mathematical ability was obtained by use of an investigator-designed pre-test of mathematical skills and knowledge prerequisite to study of the units of quadratics in Mathematics 20. Data on student achievement and retention as a result of both methods was obtained by use of investigator-designed tests of content covered during the experiment. The population of the Mathematizing Mode was 147 students on all three tests, while the population of the Expository Mode was 154 students on all three tests.

Outline of the Report

The present chapter is an outline and preview of the study. Chapter II consists of a review of literature relating to the study. Chapter III contains a complete description of the design of the experiment and preparation of the instruments. Chapter IV consists of the results from the statistical analysis of the data gathered,

while Chapter V consists of a summary of the investigation and a discussion of possible conclusions and limitations, together with recommendations for further research and educational practice.

CHAPTER II

REVIEW OF RELATED LITERATURE

Outline of Chapter

After an introduction, the present chapter first sets forth some modern-day objectives of mathematics instruction. Then the point is made that a particular teaching method is suitable for the realization of a particular objective. Next, literature is reviewed to establish that discovery and expository teaching are related to different instructional objectives. Following this, a rationale is developed for the idea that different teaching methods dictate different evaluative procedures. Thus, discovery teaching demands evaluative procedures different from those of expository teaching. Then, studies which employed different evaluation for discovery teaching as opposed to expository teaching are reviewed. Next, studies which employed the same evaluation for discovery and expository teaching are reviewed. The chapter concludes with a summary of the literature reviewed, followed by conclusions derived and difficulties encountered in this type of research, together with a statement explaining how the present study attempts to overcome these difficulties.

Introduction

The present study was a comparison of discovery and expository teaching. It was conducted in the field of college-preparatory mathematics at the high school level. A variety of discovery teaching, called the Mathematizing Mode, was used, and a variety of expository

teaching, called the Expository Mode, was used. The prime purpose of this study was the comparison of student achievement as a result of the Mathematizing Mode and the Expository Mode.

A survey of the literature on discovery learning soon revealed the fact that no common definition of "discovery" exists. The same may be said of "expository". Therefore, the representative studies presented here will be those which utilized discovery and expository modes considered to be similar in at least some respects to the modes used in this study. That is, studies reviewed will compare discovery teaching with expository teaching; the subject area will be mathematics, and the discovery and expository teaching methods will at least be related to the modes defined for this study.

Objectives of Mathematics Instruction

As stated previously, emphases on the objectives of mathematics instruction are constantly shifting. Currently, the major common goals of mathematics teaching are well presented by Johnson and Rising:

> A. The student knows and understands concepts such as mathematical processes, facts, or principles.
>
> B. The student understands the logical structure of mathematics and the nature of proof.
>
> C. The student performs computations with understanding, accuracy, and efficiency.
>
> D. The student has the ability to solve problems.
>
> E. The student develops attitudes and appreciations which lead to curiosity, initiative, confidence and interests.
>
> F. The student learns how to develop proper methods of learning mathematics and communicating mathematics, and also develops study habits essential for independent progress. (Johnson and Rising, 1967, p. 12 - 14)

The above objectives are present-day attempts of the mathematics educator to meet the demands of society. These demands insist that "the mathematics program must produce students who know how to learn mathematics, enjoy learning mathematics, and are motivated to continue their learning." (Johnson and Rising, 1967, p. 11) This represents a swing in emphasis from a concentration on manipulative techniques and skills towards a greater understanding of the bases for these techniques and skills, toward problem solving skills, and toward the processes by which mathematics develops.

The foregoing statements indicate that the objectives of mathematics instruction with their shifting emphases place a tremendous task before the mathematics teacher.

Objectives Related to Methods

The varied objectives of mathematics instruction place great demands on the mathematics teacher. He must be master of many teaching techniques and he must know when to use them. To again quote Johnson and Rising:

> The teacher of mathematics must know when and what concepts to teach, when and why students are having difficulty, how to make concepts meaningful, when and how to practice skills, and how to stimulate productive thinking. The current emphasis on discovery, problem solving, and attitudes poses problems of adaptability and flexibility in the classroom -- problems that require far greater skill than that required for the usual day-to-day lecture-recitation presentation. (Johnson and Rising, 1967, p. 7)

In summary, it is easily accepted that no one teaching method will suffice for the realization of all the many and varied mathematics education objectives. Thus, it seems obvious that a particular teaching method may be more suitable for the realization of a particular objective

than is a different instructional technique. This seems to be the case with discovery teaching and expository teaching.

The Objectives of Discovery and Expository Teaching

B.Y. Kersh

In his article "Learning by Discovery: What is Learned" (Kersh, 1964), Kersh indicates, after a survey of results of discovery teaching as reported in research literature, that perhaps discovery teaching is useful in meeting educational objectives different from those of expository teaching. With respect to discovery teaching, he states:

> To summarize the claims for learning by discovery, then, we may say this: the claim is that when the student learns by independent discovery he (a) develops an interest in the task, (b) understands what he learns and so is better able to remember and to transfer what is learned, and (c) learns something the psychologist calls a 'learning set' or a strategy for discovering new generalizations. (Kersh, 1964, p. 227)

Speaking of the possibility that students learn different things as a result of expository or directed teaching, Kersh further states:

> The learner may acquire more effective ways of problem solving through the discovery process than through another process simply because he has an opportunity to practice different techniques and because his more effective techniques are reinforced. Similarly, the learner may become more proficient in applying rules through the directed process of teaching simply because, through formal practice, he has more opportunity for effective practice and reinforcement than otherwise. (Kersh, 1964, p. 231)

There seems little doubt that the student learns different things as a result of discovery teaching as opposed to expository or directed teaching.

Discovery learning would seem to be related to motivational,

understanding and problem-solving objectives of mathematics instruction. (Perhaps discovery methods are aimed at objectives A, D, E, F, quoted on p. 13 from Johnson and Rising.) Expository learning, or to use Kersh's term, "directed learning", would seem to be related to manipulative, rule-stating, or rule-applying objectives. (Perhaps expository methods are aimed at objectives A, B, and C, quoted on p. 13 from Johnson and Rising.) Of course it must be realized that there is a great deal of overlap between these objectives and the emphasis placed on them. However, it seems safe to conclude from Kersh's review of recent research (Kersh, 1964) that discovery and expository learning are definitely related to different instructional objectives. These objectives may be different only in emphasis, but are then still different enough to make different evaluation procedures necessary.

Methods Related to Evaluation

Any teacher of mathematics is well aware that the term "student achievement" means different things to different people. The usual idea brought to mind by this expression is a student's score on an examination which is supposed to measure the degree of "mastery" of the subject matter. Thus achievement usually depends, at least in part, on an examination. By changing an examination, one can alter student achievement.

It is an accepted fact from curriculum theory that different instructional objectives require different evaluative procedures. It has been established earlier that different objectives dictate different teaching methods. Thus, it seems safe to assume that different teaching methods could involve different evaluative procedures,

that is, probably different examinations. This seems to be true in the area of discovery teaching as opposed to expository teaching; that is to say, student achievement as a result of discovery teaching could be evaluated differently from student achievement as a result of expository teaching. One could use a different examination of student achievement after discovery teaching than after expository teaching, simply because the objectives of discovery teaching are different, at least in emphasis, from those of expository teaching. This has been done in the studies by Cummins and Sobel, the reviews of which follow later. (Cummins, 1960; Sobel, 1956)

In the majority of studies reviewed by the writer, the above distinction in evaluation of student achievement as a result of discovery and expository teaching was not made. That is, student achievement as a result of the two treatments was measured by a common instrument. This seems to be the case in the studies by Meconi, Price, and Worthen reviewed later. (Meconi, 1967; Price, 1967; Worthen, 1967)

It must be pointed out that where one student achievement instrument is employed to measure the effects of two different teaching methods which may produce different outcomes, the possibility that the common instrument may favour one teaching method or the other cannot be overlooked. One possibility holds promise for overcoming this difficulty. That possibility is a "standard" achievement test capable of measuring student "mastery" of subject matter on many, or nearly all, possible dimensions. How does one construct such a test?

A committee of college and university examiners classified with precision and clarity educational objectives and test items for all subjects in the Taxonomy of Educational Objectives, commonly

known as Bloom's Taxonomy. (Bloom, 1956) Three divisions of educational objectives -- cognitive, affective, and psychomotor -- were defined. Objectives which relate to the recognition and recall of knowledge and the development of intellectual skills and abilities were included in the Cognitive Domain. It is generally felt that the majority of instructional objectives relating to student achievement for any mathematics program fall into the Cognitive Domain. Thus, the possibility exists that an examination of student achievement based on Bloom's Taxonomy will be a fair examination of student achievement, regardless of the methodology employed by the teacher. An examination based on Bloom's Taxonomy has the possibility of being that "standard" achievement test by which student achievement can be measured as a result of discovery teaching or expository teaching, without bias towards either method.

Studies in Which Student Achievement as a Result of Discovery and Expository Treatments is Evaluated Separately

K. Cummins

In a study where student achievement was compared on the basis of two tests, one for the discovery group and one for the expository group, Cummins dealt with freshmen calculus students at the college level. The students were divided into two comparable groups on the basis of a pre-test in calculus, a psychological test, and average grade in previous mathematics courses. One group was taught first-quarter calculus using a method of discovery very similar to the Mathematizing Mode. The other group was taught the same material in the usual college lecture-fashion by experienced teachers.

At the end of the course, both groups wrote two tests: Test One designed for the discovery or experimental group; Test A designed for the traditionally taught group. Test One was characterized by short answer discussion questions designed to measure understanding. It also included some problems. Test A was mainly problems involving more routine manipulations and applications. The following table summarizes Cummins' results.

ARITHMETIC MEANS OF SCORES ON TESTS

Section taking test	Score on Test One made for Experimental Section One Total possible score 272	Score on Test A made for Traditional Section A Total possible score 180
Section One (Experimental)	217	130
Section A (Traditional)	131	130

(Cummins, 1960, p. 167)

The scores on Test One for the discovery group were significantly higher than the scores for the traditional group. There was no significant difference in scores on Test A.

These results would seem to

> . . . indicate that the method of teaching under examination was especially effective in promoting a deeper understanding of calculus and that this gain was not at the sacrifice of proficiency in manipulations and applications. (Cummins, 1960, p. 168)

M.A. Sobel

In a study primarily concerned with concept learning, seven ninth grade algebra classes were taught using a defined discovery method, and seven classes were taught using a defined expository method. Each class was taught by a different teacher, with the exception of one instance where a teacher taught two experimental, that is, discovery, classes. Both experimental and control (expository) teachers were given written instructions concerning possible activities to be included in their classes. The material taught included concepts of general number, formula, exponent, and coefficient as they appear in a beginning course in algebra.

After a four-week period, evaluation of the experiment was made by two parallel forms of a test of two parts: Part I of the test dealt with concepts; Part II dealt with fundamental skills. Presumably, Part I was designed to favour the discovery group, while Part II was designed to favour the expository group. One form of the test administered immediately after the experiment showed no significant difference between the experimental and control groups in achievement for students of average IQ. There was a significant difference in achievement favouring the experimental group (discovery) for students of high IQ on both the concept and fundamental skills parts of the test.

After three months of instruction in whatever methods the teachers favoured, a second parallel form of the test was administered. The results were similar to the results of the first form of the test. That is, there was no significant difference between the experimental and control groups in retention for students of average IQ, but there was a significant difference in retention for students of high IQ on

both the concept and fundamental skills parts of the test, favouring the experimental group.

To summarize the results of this study, in Sobel's words:

> The major implication of this study is that bright students learn and retain certain concepts and skills better through an inductive, concrete, unverbalized teaching approach as opposed to a deductive, abstract, verbalized method of presentation. (Sobel, 1956, p. 429)

Studies in Which Student Achievement as a Result of Discovery and Expository Treatments is Not Evaluated Separately

L.J. Meconi

In a relatively small study, Meconi randomly divided forty-five high-ability eighth and ninth grade students into three groups. After using a common introductory program, these groups were taught number-sequence concepts using three distinct approaches: rule-and-example-given, guided-discovery, and pure-discovery. To quote Meconi:

> The results obtained after performing analyses of variance were as follows: (1) All three approaches led to learning in terms of measures of time and errors. (2) There were no significant differences in the approaches on a problem-solving test of immediate transfer and on a retention test given four weeks later. (3) The pure-discovery approach was the most effective as far as time taken to learn and to solve problems was concerned. (Meconi, 1967, p. 863)

J. Price

In another small study concerned with transfer of training, as well as discovery and expository teaching, Price used three grade ten general mathematics classes. One class was taught using a discovery method, another using an expository method, and the third was taught using a discovery method, and, in addition, used materials designed to promote transfer of training. Students were assigned

randomly to the experimental classes. A pre-test was administered to determine achievement in mathematics among other things. All classes met for the same length of time and no one class received additional attention.

> From the results of the testing program, the following conclusions were drawn.
>
> The two groups taught mathematics by techniques designed to promote student discovery of concepts:
> (a) showed a slight but statistically non-significant gain over the control group in achievement in mathematics;
> (b) showed a greater increase in mathematical reasoning than did the control group; and
> (c) showed a positive attitude change toward mathematics, while the control group showed a negative change. (Price, 1967, p. 875)

B.R. Worthen

Worthen's large study involved four hundred and thirty-two fifth and sixth grade pupils in sixteen classes in eight schools. He compared two teaching methods, one a discovery method, the other an expository method. His discovery method, called Treatment D in the study, is briefly described: "Verbalization of each concept or generalization is delayed until the end of the instructional sequence by which the concept or generalization is to be taught." (Worthen, 1967, p. 46) His expository method, called Treatment E in the study, was "a method in which verbalization of each concept or generalization is the initial step in the instructional sequence by which the concept or generalization is to be taught." (Worthen, 1967, p. 46) Worthen hypothesized that Treatment D would produce superior results to Treatment E on each of the criterion measures, including student achievement.

Two classes in each of eight schools were involved in the

experiment. One class was the discovery class, the other the expository. The same teacher taught both classes after exposure to a training program designed to acquaint teachers with both treatments. The classes were quite comparable on measures of IQ, arithmetic computation skill, arithmetic problem-solving ability, prior knowledge of mathematical concepts, attitudes towards arithmetic, and pupil perception of teaching behavior.

A definite attempt was made to eliminate, or at least control, the Hawthorne effect, in that the same teacher taught both classes and both treatments lasted the same length of time.

The mathematical concepts taught to both groups were:

> (1) notation, addition, and multiplication of integers (positive, negative, and zero); (2) the distributive principle of multiplication over addition; and (3) exponential notation and multiplication and division of numbers expressed in exponential notation. (Worthen, 1967, p. 49)

After a six-week experimental period, student achievement was measured by a Concept Knowledge Test. A parallel form of this test, the Concept Retention Test was administered twice to both treatment groups, once five weeks after instruction and once eleven weeks after instruction, to measure retention.

The results of the experiment as regards to student achievement are best stated by Worthen:

> The data yielded by the Concept Knowledge Test did not support the hypothesis that Treatment D would produce superior results on an initial learning test. On the contrary, these data showed Treatment E to produce significantly better results than Treatment D on the initial learning criterion test. . . .
>
> The hypothesis that Treatment D would produce superior results to Treatment E on a retention test given five and eleven weeks after instruction was supported by the evidence yielded by an analysis of the Concept Retention Test scores. (Worthen, 1967, p. 54)

A summary of Worthen's results regarding student achievement appear in the following table. It is significant to note that the expository group achieved higher scores on the immediate post-test, but the discovery group achieved higher scores on the retention tests.

SUMMARY OF ANALYSES OF COVERAGE OF CRITERION MEASURE POST-TEST SCORES: BETWEEN TREATMENTS D AND E

Measure	df_1	df_2	F	P	Direction
Concept Knowledge Test	1	412	7.435	$<.01$	$D<E$
Concept Retention Test 1	1	412	3.918	$<.05$	$D>E$
Concept Retention Test 2	1	412	5.868	$<.025$	$D>E$

(Worthen, 1967, p. 55)

Summary

Theoretical Considerations

On the theoretical level, proponents of discovery, exemplified by Bruner, insist that discovery learning is a personal way of learning that is highly motivating and stimulating. Those who are less enthusiastic about discovery, exemplified by Ausubel, insist that it is time-consuming and inefficient and inappropriate for certain age levels. It cannot be used all the time for every purpose. There is agreement among many scholars that discovery is a valuable teaching method, but its most effective role in mathematics education has yet to be determined.

A consideration of the various outcomes of discovery and expository learning cited in research has led writers, such as Kersh,

to bring forth the important possibility that perhaps discovery learning is related to different instructional objectives from those of expository learning. This is perhaps part of a much larger problem raised by Cronbach, who states:

> Only gradually are we moving away from the experimental paradigm in which amount or rate of learning is the sole dependent variable, and into a timid attempt to appraise educational development multidimensionally. (Cronbach in Shulman and Keislar, 1966, p. 88)

It seems that Cronbach and others, such as Wittrock (Wittrock in Shulman and Keislar, 1966), are saying that since there are so many outcomes of teaching by any particular method, it just doesn't make sense to compare two methods on the basis of one or two variables. One method, for example, discovery, may produce better results in one area of educational outcomes, say the area of problem solving or motivation or student attitudes. Another method, for example, expository, may produce better results in another area of educational outcomes, say the area of manipulative or fundamental skills.

In the light of the afore-mentioned possibility that discovery teaching may have greater value in certain areas of educational outcomes than does expository teaching, it should again be noted that the present study was part of a complete group project. This project attempted to compare discovery and expository teaching outcomes on the basis of several dimensions -- student achievement, student attitudes and reactions, and student creativity among others.

Research Findings

A survey of the research on discovery learning as opposed to expository learning leads to very little in the way of conclusive

evidence. This is substantiated by Wittrock, who says: "The current state of research on discovery is very disappointing and precludes any important conclusions about teaching or learning." (Wittrock in Shulman and Keislar, 1966, p. 45) There are many problems which contribute to the unsatisfactory state of research in discovery versus expository teaching situations. One of these is the semantic difficulties that arise when referring to discovery. As Wittrock says: "The rote (expository) treatment group of one study is sometimes indistinguishable from a discovery treatment of another study." (Wittrock in Shulman and Keislar, 1966, p. 44)

Another problem is that little seems to have been done to determine which objectives of mathematics learning are best achieved by discovery methods and which by expository methods, and to evaluate learning or effectiveness in terms of these.

It should be recalled that the basic purpose of this study was the comparison of student achievement as a result of discovery and expository teaching. A short summary of the results of the representative studies reviewed earlier, with respect to student achievement, follows.

In the Cummins' study, a group of freshmen college students, that is, students of relatively high ability who were taught in a discovery mode, achieved significantly more on a test measuring understanding than a comparable group who were taught in an expository mode, and achieved as well on a fundamental skills test as did the expository group.

In the Sobel study, grade nine mathematics students of average

ability, who were taught as a discovery group, achieved as well as, but not better than, a comparable expository group on both parts of a test, one part favouring the discovery group, the other part favouring the expository group. For students of high ability, the discovery group performed significantly better than the expository group. These results for both average and high ability groups were repeated on a retention test of student achievement.

The Price study, which dealt with grade ten general mathematics students, possibly with students of lower ability, revealed that the discovery group achieved slightly (statistically non-significantly) better than a comparable expository group on an achievement test and on a test of mathematical reasoning.

Meconi found that a discovery group of grade eight and nine high ability students achieved no better results on an achievement test than did a comparable expository group. He did find, however, that the discovery group mastered the material taught in less time than the expository group.

In his large study at the elementary level, Worthen found that the discovery group comprised of pupils of all ability levels did not achieve as well on an achievement test as did a comparable expository group. However, there was a startling reversal of form in his results on a retention test, which showed the discovery group achieving more than the expository group.

From the foregoing summary of representative literature, one can easily see that definite conclusions as to superior student achievement as a result of discovery or expository teaching cannot be drawn. While the discovery groups tend to achieve better results

in achievement in some studies, this may be attributable to many circumstances. One tentative conclusion that may be reached is that retention of concepts may be helped by use of a discovery method. This seems to be borne out at least in the Worthen study.

However, in general, with respect to student achievement as a result of discovery and expository teaching, one must agree with Meconi who says: "No definitive conclusions concerning studies and experiments of expository versus discovery approaches of instruction in mathematics can be drawn from the literature." (Meconi, 1967, p. 862)

Conclusions

The literature as reviewed makes the following points on a theoretical level:

1. There are those who feel that the discovery approach is somewhat of an educational panacea -- that it is the way that students of all ages ought to learn almost everything that they are going to learn in formal education. This view is opposed by others who feel that the discovery approach has only limited value to students at a certain stage of development and in particular subject areas.
2. The possibility has been raised that perhaps the discovery approach is suitable for meeting educational objectives different in nature from those for which the expository approach is suited; that is, the discovery approach is related to certain objectives while the expository approach is related to other objectives.

On a research level, a call has been sounded for more research of the discovery versus expository type which is multi-dimensional; that is, more large studies are needed in which the merits of the discovery approach as opposed to the expository approach are examined on a large variety of dimensions. Comparisons of one or two facets of instruction drawn in isolation may lead to erroneous conclusions.

The foregoing review of representative studies comparing discovery and expository modes of instruction also revealed several difficulties encountered by research of this type.

One of these difficulties is the lack of control of the Hawthorne effect. Very often, the control group, that is, the expository group, is given no experimental treatment. These classes appear to be conducted in a "status quo" fashion, the nature of which has seldom been defined. The expository methods are rarely defined as proper experimental methods and little effort has been made to incorporate a well-structured expository mode. This problem is recognized by Bittinger, who says:

> In a review of the research on discovery one problem stands out. The didactic (teacher sets forth knowledge) method has not been given a fair chance in the control group. The non-discovery group should be given the same well-tuned effort as the discovery group. (Bittinger, 1968, p. 142)

When superior student achievement is obtained using the discovery treatment, it may possibly be attributed to this cause.

Another difficulty that is common to research of this type is the fact that the discovery and expository methods employed are not carefully defined. At times, the expository method of one study is indistinguishable from the discovery method of another study. This claim is substantiated by Worthen, who found that there was

no definite order of learning tasks set out for either the discovery or expository methods used in the reported research.

A third difficulty common to this type of research involves the evaluation of student achievement. This is most often by means of a common examination for both groups. Usually, very little is said about this examination. Thus, it is not known whether the test favours one method or the other in terms of unstated objectives.

The inconclusive and conflicting results of studies comparing discovery and expository methods of instruction and the difficulties associated with these studies indicate that no definite conclusion can be drawn regarding the effectiveness of the two modes for promoting student achievement in mathematics.

The present study attempts to overcome the difficulties associated with other studies by:

1. being a part of a large project which compared discovery and expository teaching on many dimensions;
2. making the expository group definitely an experimental group;
3. carefully defining the discovery and expository methods employed in terms of student and teacher activities;
4. evaluating student achievement by means of a common examination which sampled several levels of educational objectives as stated in Bloom's Taxonomy. This was an attempt to cover all desired objectives of instruction regardless of teaching method.

The present study was an attempt to add objective evidence

from which it will be possible in the future to determine the value and place of discovery teaching in secondary mathematics education, particularly at the senior high school level.

CHAPTER III

THE EXPERIMENTAL DESIGN AND THE STATISTICAL PROCEDURES

Introduction

The present study was part of a large group project which compared discovery teaching with expository teaching. Many facets of discovery opposed to expository teaching were compared, among them student achievement, student attitudes, teacher behavior and student creativity. The prime purpose of the present study was the comparison of student achievement as a result of the Mathematizing Mode, a discovery teaching method, and the Expository Mode, an expository teaching method. The population consisted of grade eleven students from the Edmonton Public School System, and the material taught by the two methods under consideration was quadratic functions and equations as found in a senior matriculation or college preparatory high school mathematics course.

A detailed description of the group project follows.

Design of the Experiment

The original impetus for the group project, concerned with discovery versus expository teaching, came out of a 1967-68 University of Alberta graduate mathematics education seminar. Realizing that discovery teaching and learning was an important area of mathematics education research, four graduate students, one of whom was the writer, accepted responsibilities to develop discovery teaching materials and to investigate student achievement, student attitudes and creativity, and teacher behaviors in a discovery versus expository teaching project.

The present study focusses on the student achievement aspect of the above project.

During the fall term of the University of Alberta 1967-68 academic year, approval of the Edmonton Public School Board to carry out the project was obtained. Simultaneously, discovery materials were developed by another investigator, Johnston. (Johnston, 1968) He defined several distinct characterizing activities which comprised the Mathematizing Mode, a discovery teaching method explained in Chapter I, page 6. A pilot study, using material on the linear function in the Mathematizing Mode, as well as on the quadratic function, was conducted with a class of high school mathematics students similar to those anticipated to be involved in the later project. During the pilot study, the activities and materials were refined and modified until they eventually comprised the Mathematizing Mode.

The expository method of teaching, called the Expository Mode, was similar to the traditional lecture form of teaching mathematics at the high school level. However, the Expository Mode, as found in Chapter I, pages 6, 7, was carefully defined in terms of activities and materials by a mathematics education professor.

The subject matter chosen to be taught to both treatment groups throughout the project was chapters nine and ten of the grade eleven Mathematics 20 Alberta textbook. The chapter titles are: "The Quadratic Function and its Applications" and "Introduction to The Theory of Quadratic Equations".[6]

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[6] See Appendix C for a section-by-section description of the material covered in these chapters.

The co-operation of teachers who taught two classes of Mathematics 20 was solicited. Nine teachers in six different Edmonton high schools expressed an interest in participating in the project. All nine teachers attended in-service sessions designed to acquaint them with the Mathematizing Mode and the Expository Mode. The objective of the in-service sessions was to prepare teachers to exhibit the behaviors required for the Mathematizing and Expository Modes when teaching the Mathematizing and Expository classes respectively. The in-service sessions, conducted by two Mathematics Education professors before and during the actual project, consisted of informal discussions and explanations of teacher behaviors expected while employing the two teaching methods.

Before the actual experiment began, the Mathematizing classes were exposed to a "warm-up" period, during which teachers "practiced" the Mathematizing Mode on a small unit on the linear function.

Of the nine teachers involved in the project, seven taught at least two Mathematics 20 classes. These seven teachers were asked to teach one class in the Mathematizing Mode and the other class in the Expository Mode. The remaining two teachers, herein labelled Teacher A and Teacher H, had only one class of Mathematics 20. Teacher A taught his class in the Mathematizing Mode and Teacher H taught his class in the Expository Mode throughout the duration of the project. This continued throughout the months of February and March, 1968. All teachers were observed throughout the project by an investigator, to determine to what extent their behaviors in a Mathematizing class actually differed from those in an Expository class.[7] At the conclusion

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[7] See Appendix B for relevant results.

of the two units on quadratics, an achievement post-test was administered to both treatment groups. A parallel form of the achievement post-test was administered to both groups approximately seven weeks after the conclusion of the treatments.

In addition, other investigators involved in the project administered student questionnaires to determine student attitudes and reactions to the Mathematizing Mode. Interviews were conducted with a subset of the student population to ascertain student opinions of teacher behaviors exhibited during the Mathematizing Mode, and of the Mathematizing Mode itself. Creativity pre-tests and post-tests were administered to a subset of the student population as well.

No attempt was made to govern the length of time the teachers took to cover the material. The only criterion established was that the teachers feel certain that their classes have mastered the materials involved. Due to varying ability of classes and different school schedules, not all teachers took the same length of time to complete the material. The shortest time taken by any one teacher to cover the material was 6 weeks, while the longest time taken was 12 weeks.

It must also be mentioned that all classes involved in the project realized that they were in an experiment. The Expository Mode was considered just as experimental by the teachers as the Mathematizing Mode. Teachers' Expository classes were observed just as frequently as were their Mathematizing classes. Thus it was hoped to neutralize the Hawthorne Effect.

As previously mentioned, nine teachers in six Edmonton high schools participated in the project. The six schools represented a wide

variety of school populations. One school's population enjoyed superior socio-economic home conditions, while three schools' populations were not quite so well favoured, and two schools' populations were from poorer socio-economic backgrounds. Thus, the sample in the project consisted of students whose home background was representative of practically the whole spectrum of socio-economic conditions.

For the seven teachers who each taught at least two classes of Mathematics 20, the assignment of class to treatment was random. Teachers were asked to choose two classes that were "alike" in achievement, and then treatment was randomly assigned.

The total population involved in the experiment consisted of four hundred and ninety-three grade eleven Mathematics 20 students in sixteen classes. Eight classes were taught in the Mathematizing Mode and eight in the Expository Mode. Since it was deemed desirable to deal only with groups that were exposed to both treatments by the same teacher, the classes of teachers A and H were eliminated from the sample. It also was considered desirable to deal only with students who wrote the pre-test, post-test and retention test. The population was again limited in this regard. These considerations reduced the total number of students in the experiment for comparison purposes to three hundred and eighteen, with one hundred and forty-seven in the Mathematizing Group (M) and one hundred and seventy-one in the Expository Group (E).

Development of Testing Instruments

As previously mentioned, three tests of student achievement were administered to both groups; a pre-test, post-test and retention

test. These instruments were prepared by the writer as no suitable standardized test covering the content taught before and during the experiment was available. This present section consists of a description of the development of these testing instruments.

Relation of Tests to Bloom's Taxonomy

In Chapter II, a rationale was developed for the use of examinations based on Bloom's Taxonomy. It was hoped that an examination of this type would not favour either treatment group while covering all desired cognitive objectives of instruction, thus providing a firm standard of measurement of student achievement. All three instruments used in this study were developed on the basis of Bloom's Taxonomy. The table that follows is a summary of Bloom's classifications of educational objectives in the Cognitive Domain.

Knowledge

1.00 Knowledge

- 1.10 Knowledge of Specifics
  - 1.11 Knowledge of terminology
  - 1.12 Knowledge of specific facts
- 1.20 Knowledge of Ways and Means of Dealing with Specifics
  - 1.21 Knowledge of conventions
  - 1.22 Knowledge of trends and sequences
  - 1.23 Knowledge of classifications and categories
  - 1.24 Knowledge of criteria
  - 1.25 Knowledge of methodology
- 1.30 Knowledge of the Universals and Abstractions in a Field
  - 1.31 Knowledge of principles and generalizations
  - 1.32 Knowledge of theories and structures

Intellectual Skills and Abilities

2.00 Comprehension

- 2.10 Translation
- 2.20 Interpretation
- 2.30 Extrapolation

3.00 Application
4.00 Analysis
    4.10 Analysis of Elements
    4.20 Analysis of Relationships
    4.30 Analysis of Organizational Principles
5.00 Synthesis
    5.10 Production of a Unique Communication
    5.20 Production of a Plan or Proposed Set of Operations
    5.30 Derivation of a Set of Abstract Relations
6.00 Evaluation
    6.10 Judgment in Terms of Internal Evidence
    6.20 Judgment in Terms of External Criteria

It must be remembered that Bloom's categories are not mutually exclusive since the higher levels generally include the lower levels. This seems to be quite logical in the case of educational objectives because a person cannot perform at a high level of thought without involving the lower levels. To comprehend, a person must also know; to apply, he must first know and comprehend. As a consequence, test items are classified according to the highest level of thought required.

The criterion for deciding in which category a test item should be placed is the thought process used by the pupils to arrive at the correct response. Classification, then, must be made on the basis of the average student in the class because different pupils will no doubt use different thought processes in responding to an item. For instance, some pupils might use application in dealing with a certain item because for them the situation is new; other students might be familiar with the task and use only knowledge in responding. Classification of test items, therefore, is relative to the reader's view of what thought processes will be used by most pupils in responding to the item.

The following Table I summarizes the writer's attempt to

classify the items included in the pre-test, post-test and retention test according to Bloom's Taxonomy.[8]

TABLE I

NUMBER OF ITEMS INCLUDED IN BLOOM'S CATEGORIES

	Pre-test	Post-test	Retention Test
1. Knowledge	5	0	0
2. Comprehension	5	0	0
3. Application	16	15	17
4. Analysis	1	13	11
5. Synthesis	1	2	2
6. Evaluation	1	0	0

Description of Pilot Studies

Very few examinations of student mastery of subject matter are satisfactory after initial writing of the items. Most good tests are revised and improved several times before evaluation experts consider them usable. An attempt was made to improve each of the three instruments used in this study through use of a pilot study.

Initially, the writer wrote test items which he considered satisfactory on two counts: (1) the items sampled at least several

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[8] See Appendix D for copies of the three tests.

of Bloom's categories; and (2) the over-all test was somewhat typical of a classroom teacher's test, that is, the sort of test that would be familiar to the students in terms of format and difficulty level. Then, the items were shown to a Mathematics Education professor for approval. Next, the tests were administered to a small pilot population of students comparable to those involved in the project. The purpose of the pilot studies of the tests was to check on the tests' reliability and to help in the identification of poor items. An item analysis was performed on each pilot examination. The examinations were then improved by discarding poor items (generally those with a difficulty index below .30), by changing poor distractors and by rearranging the order of the items. The following Table II summarizes the results of attempts to improve the examinations. These reliability coefficients are taken to indicate sufficiently high reliability for the measures used in this study.

TABLE II

MEANS AND RELIABILITIES OF PILOT STUDY AND PROJECT TESTS

Test	No. of students	Total possible score	Mean	Kuder-Richardson 20 Reliability
Pre-test Pilot	104	30	18.31	.59
Post-test Pilot	100	28	15.14	.74
Retention Test-Pilot	65	30	16.54	.61
Pre-test	485	29	18.85	.68
Post-test	493	30	14.49	.71
Retention Test	383	30	16.30	.73

Testing Program

Description of Tests

The pre-test consisted of twenty-nine multiple choice items based on material covered by the students in the project prior to the material taught during the project. The pre-test was based on Chapters I to IV in the Mathematics 20 text, "Secondary School Mathematics", Grade Eleven, Edition 2, by Beesack, MacLean et al. The material in these chapters consists of: Chapter I: Real Numbers, Equations and Inequations; Chapter II: Exponents, Square Roots,

Radicals; Chapter III: Introduction to Binary Relations; Chapter IV: Functions.

The post-test consisted of thirty multiple choice items based on the material covered throughout the project. This material is outlined in Appendix C.

The retention test consisted of thirty multiple choice items based on the same material as the post-test. In fact, an attempt was made to make the retention test a parallel form of the post-test by constructing the items in the retention test as similar to the items in the post-test as possible.

The tests themselves are found in Appendix D.

Description of Test Administration

All three tests were administered by the teachers involved in the project. Every teacher was supplied with the same list of instructions as to procedure to be followed in administration of the tests. Student responses to items on all tests were recorded on machine-scoring answer sheets, supplied by the writer. Because class length time varied slightly from school to school, all three examinations were limited to forty minutes in length, to keep this standard for all students in the testing program.

A pre-test was administered to the students immediately before the beginning of the experimental period which lasted for approximately seven weeks. The post-test was administered immediately after the experimental period, and the retention test was administered approximately seven weeks after the post-test.

All examinations, including the pilot study tests, were

marked by an IBM Optical Scorer.

Treatment of Data

Source of Data

All student achievement scores were obtained from the investigator-prepared examinations previously described. All intelligence quotients were obtained from existing school records. In all cases, a Lorge-Thorndike non-verbal IQ measure was used. Students for whom this measure was unavailable were not included in the data sample.

Statement of Analysis Used

The prime consideration was to test for mean differences between the Mathematizing and Expository groups. The groups were each divided into three IQ levels: Lo, IQ's of 90-115 inclusive; Average, IQ's of 116-125 inclusive; and Hi, IQ's of 126-150 inclusive. This was done so that the effects of the two different treatments on students of three different ability levels could be assessed. The number of students classified in each group and IQ level, IQ means and standard deviations for these groups are indicated in Table III.

TABLE III

CELL FREQUENCIES, MEANS AND STANDARD DEVIATIONS FOR INSTRUCTIONAL GROUPS BY INTELLIGENCE QUOTIENT LEVEL CLASSIFICATION

Group	Intelligence Quotient			
	Lo	Ave	Hi	
Mathematizing (M)	49	51	47	Number
	107.26	120.75	131.90	Mean
	6.17	2.65	3.97	Standard Deviation
Expository (E)	56	47	68	Number
	107.41	120.83	133.03	Mean
	7.17	2.97	5.7	Standard Deviation

Null Hypotheses Tested

1. There is no significant difference between the mean pre-test scores obtained by the M and E groups.
2. There is no significant interaction between modes of instruction and level of intelligence on the post-test.
3. There is no significant difference between the mean post-test scores obtained by the M and E groups.
4. There is no significant interaction between modes of instruction and level of intelligence on the retention test.
5. There is no significant difference between the mean retention test scores obtained by the M and E groups.

The hypotheses were tested by a two-way analysis of variance using program ANOV 25 of the Division of Research Services of the

University of Alberta. This program tests for interaction and gives two solutions for analysis of variance: one based on a no interaction assumption; the other, without this assumption. It also produces a means and a variance matrix together with a chi-square test for homogeneity of variance.

Summary

The present chapter has been a description of the nature of the various parts of the project, of the experimental design and the development of the instruments, and of the statistical procedures to be used to analyze the data. The next chapter presents the results of the analysis of the data.

CHAPTER IV

THE RESULTS OF THE INVESTIGATION

Introduction

This study compared the two variants of the independent variable: teaching method -- the Mathematizing Mode, a discovery method and the Expository Mode, an expository teaching method, with respect to student achievement as a dependent variable. The writer produced three examinations of student achievement based on Bloom's Taxonomy; a pre-treatment test, a post-treatment test, and a retention test. These examinations were administered to all students in the project before, immediately after, and seven weeks after a seven-week experimental period during which the seven teachers involved in the project each taught one Mathematics 20 class in the Mathematizing Mode and another Mathematics 20 class in the Expository Mode.

The present chapter offers the results of the examinations and a statistical comparison of these results for the Mathematizing group and Expository group. The matrix immediately following demonstrates the system of analysis used.

TABLE IV

NUMBER OF OBSERVATIONS MATRIX

A. Treatment Groups	B. Intelligence Levels		
	Lo	Ave	Hi
M	49	51	47
E	56	47	68

In the analysis which follows, "A" represents the Treatment Groups; Mathematizing group (M) and Expository group (E). "B" represents the Intelligence Levels; low IQ category (Lo), average IQ category (Ave), and high IQ category (Hi).

Procedure

The following system of reporting the results of the examinations is employed in the pages that follow. First, a statement of the null hypothesis to be tested is presented, followed either by a matrix of means and variances with the same cell arrangement as in Table IV, or a table giving the results of an interaction test. (The means and variances relate to the appropriate examination.) Next, a table containing the analysis of variance data is presented, if appropriate. Conclusions are then stated concerning the rejection or non-rejection of the null hypothesis for either interaction or main effect.

Comparison of Groups Before Treatment

Null Hypothesis I: There is no significant difference between the mean pre-test scores obtained by the M and E groups.

TABLE V

MEANS AND VARIANCES OF GROUPS ON PRE-TEST

A. Treatment Groups	B. Intelligence Levels			
	Lo	Ave	Hi	
M	18.67	19.02	20.47	Mean
	11.89	16.86	19.12	Variance
E	18.68	19.57	21.32	Mean
	20.84	21.99	16.84	Variance

TABLE VI

ANALYSIS OF VARIANCE - PRE-TEST

Source	Sum of Squares	D.F.	Mean Squares	F-ratio	Probability
A	17.9184	1	17.9184	1.006506	0.3165
B	302.066	2	151.033	8.483788	0.0003

On the basis of the information in the A row of Table VI, Null Hypothesis I was not rejected.

Comparison of Groups After Treatment

Null Hypothesis II: There is no significant interaction between modes of instruction and level of intelligence on the post-test.

TABLE VII

TEST FOR INTERACTION BETWEEN A AND B ON POST-TEST

Source	Sum of Squares	D.F.	Mean Squares	F-ratio	Probability
A B	16.375	2	8.1875	0.368881	0.6918

On the basis of the information in Table VII, Null Hypothesis II was not rejected.

Null Hypothesis III: There is no significant difference between the mean post-test scores obtained by the M and E groups.

TABLE VIII

MEANS AND VARIANCES OF GROUPS ON POST-TEST

A. Treatment Groups	B. Intelligence Levels			
	Lo	Ave	Hi	
M	12.78 17.47	14.00 21.96	15.74 18.15	Mean Variance
E	13.63 21.98	15.30 21.95	17.69 28.87	Mean Variance

TABLE IX

ANALYSIS OF VARIANCE - POST-TEST

Source	Sum of Squares	D.F.	Mean Squares	F-ratio	Probability
A	148.916	1	148.916	6.736389	0.0099
B	715.117	2	357.559	16.174515	0.0000

On the basis of the information in the A row of Table IX, Null Hypothesis III was rejected at the .01 level of confidence.

Comparison of Groups Seven Weeks After Treatment

Null Hypothesis IV: There is no significant interaction between modes of instruction and level of intelligence on the retention test.

TABLE X

TEST FOR INTERACTION BETWEEN A AND B ON RETENTION TEST

Source	Sum of Squares	D.F.	Mean Squares	F-ratio	Probability
A B	44.0625	2	22.0312	1.043622	0.353377

On the basis of the information in Table X, Null Hypothesis IV was not rejected.

Null Hypothesis V: There is no significant difference between the mean retention test scores obtained by the M and E groups.

TABLE XI

MEANS AND VARIANCES OF GROUPS ON RETENTION TEST

A. Treatment Groups	B. Intelligence Levels			
	Lo	Ave	Hi	
M	15.24	15.27	17.43	Mean
	13.15	20.08	16.51	Variance
E	15.48	17.21	19.13	Mean
	19.09	19.22	33.70	Variance

TABLE XII

ANALYSIS OF VARIANCE - RETENTION TEST

Source	Sum of Squares	D.F.	Mean Squares	F-ratio	Probability
A	130.249	1	130.249	6.168167	0.013543
B	517.562	2	258.781	12.255057	0.000009

On the basis of the information in the A row of Table XII, Null Hypothesis V was rejected at below the .05 level of confidence.

Summary of Findings

For both the post-test and retention test, there was no evidence to indicate that interaction was statistically significant. Hence, the results of the analysis of variance with respect to the treatment effects could be used to test the null hypotheses concerning differences of means of scores for the M and E groups.

There were no significant differences between the mean scores for the M and E treatment groups on the pre-treatment test. However, on both the post-test and the retention test, there was a statistically significant difference favouring the E group, on the former at the .01 level of confidence, and on the latter at the .05 level of confidence. Since there was no interaction, the treatment effect was considered to be the same across the three IQ levels. It might be noted that the effect appeared strongest in terms of mean differences for the high IQ group.

CHAPTER V

SUMMARY, CONCLUSIONS, DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

Summary

This study was part of a group project which compared discovery teaching with expository teaching on several dimensions. These included teacher behaviors, student creativity, student achievement and student reactions and attitudes. The purpose of this study was the comparison of student achievement as a result of the Mathematizing Mode, a defined discovery technique, and the Expository Mode, a defined expository method.

Seven teachers, each of whom taught two classes of Mathematics 20 in the Edmonton Public School System, attended in-service training sessions designed to acquaint them with the Mathematizing Mode and the Expository Mode. They taught their Mathematics 20 classes in the following way on a unit on quadratic functions: one class was taught using the Mathematizing Mode, while the other class was taught using the Expository Mode. This continued for a period of approximately seven weeks.

The writer prepared three tests of student achievement, with the items on these tests sampling several categories of Bloom's Taxonomy, Cognitive Domain. The three tests were refined through use of pilot studies. A pre-test was administered at the outset of the experiment; a post-test at the end of the experiment; and a retention

test, a parallel form of the post-test, was administered seven weeks after the post-test.

Throughout the project, three other investigators administered a variety of tests to teachers and students. All teachers were observed and rated regularly to determine if their teaching methods in their two classes were basically different. Students were interviewed and they filled out questionnaires to determine their reactions to the teaching methods employed by their teachers. Creativity tests were administered to the students as well.

Conclusions

With respect to student achievement, the essential findings were:

1. There was no significant difference between the Mathematizing and Expository groups on a pre-test of student achievement administered at the outset of the experiment.
2. There was a significant difference between the Mathematizing and Expository groups on a post-test of student achievement administered at the conclusion of the experiment. The Expository group demonstrated superior results on the post-test.
3. There was a significant difference between the Mathematizing and Expository groups on a retention test of student achievement administered seven weeks after the conclusion of the experiment. As for the post-test, the Expository group demonstrated superior results on the retention test.

In summary, it is apparent that the Expository group out-

performed the Mathematizing group on the achievement tests. However, before passing judgment on the comparative effectiveness of the Mathematizing and Expository Modes with respect to achievement, the tests on other dimensions administered during and after the project should be taken into account. The results of the complete project must be examined in order to form reasonable conclusions. On this point, one must agree with Cronbach, who says:

> Education has many purposes, and any learning experience must be judged in terms of all those goals. To take a simple example, there must exist some method of teaching arithmetic that produces graduates who compute brilliantly -- and who hate all work involving numbers. It would be improper to advocate this method on the basis of research that considers only its effect on computational skill. (Cronbach in Shulman and Keislar, 1966, p. 77)

Thus, to get a more complete picture, the general results of most tests and measures used throughout the project are presented below.

Mr. J. Vance, a fellow-investigator, was responsible for the area of student reactions and attitudes towards the two teaching methods.[9] He found that 38% of the Mathematizing group favoured continuation of the study of mathematics by the Mathematizing Mode, 48% favoured the method the teacher used before, and 14% did not care either way. Vance also interviewed students in the Mathematizing group who were outstanding either because of their enjoyment of the Mathematizing Mode or their strong dislike of the method. Typical comments ranged from: "It's been a marvellous experience" to "I hated every minute." Another point of interest noted by Vance is the fact that there seemed to be little relation between a student's

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[9] A report written by Mr. Vance, presenting the results of his investigations in this area, is found in Appendix E.

opinion of the Mathematizing Mode and his score on the post-test. In fact, in one class, the student most opposed to the Mathematizing Mode scored the highest mark on the post-test. Vance also noted that some of those students who seemed unhappy about the Mathematizing Mode expressed a belief that it was a good way to learn mathematics. They felt that it just wasn't the best way for them personally to learn mathematics.

Mr. W. Naciuk, another fellow-investigator, worked in the area of teacher-classroom behaviors before, during, and after the experiment. He found that teachers' activities definitely were different in a Mathematizing class as compared with an Expository class.[10] In addition, Naciuk administered a Post-Experiment Questionnaire to the teachers involved in the project. When asked how they personally felt about the merits of the Mathematizing Mode as compared to their personal methods, the teachers' replies were summarized by Naciuk:

> Some teachers felt that the Mathematizing method of teaching took too much class time to cover the same units. Others felt that the new experiences gained by the students and teachers were worth the extra time. (Naciuk, 1968, Appendix G)

A variety of responses were found to the question: "What do you feel the students gained from participating in the experiment?" To again cite Naciuk:

> Some teachers felt the student gained a deeper insight into mathematics when taught with the Mathematizing Method. Others noticed an appreciable increase in the students' enthusiasm for mathematics. Still others felt that the students gained from the increased classroom interaction in the mathematizing classes. (Naciuk, 1968, Appendix G)

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[10] The pertinent results in this area are found in Appendix B.

Mr. J.E.M. Taylor-Pearce, a third fellow-investigator, was interested in student creativity as a result of exposure to the Mathematizing and Expository Modes. He administered a pre-test and post-test of creativity to a sample of the population involved in the project. His tests of creativity were constructed so as to test for fluency, flexibility and originality. To quote Taylor-Pearce's results on the post-test of creativity:

> It was found that for fluency, flexibility, originality and 'total response', the treatment effects of the expository method were superior to the treatment effects of the mathematizing method. (Taylor-Pearce, 1969, p. 106)

In this present section, the writer has attempted to objectively present the general, major findings of the whole project. Briefly these were:

1. With respect to student achievement, there was little doubt as to the superior results produced by the Expository Mode as compared with the Mathematizing Mode.
2. With respect to student creativity, the Expository Mode definitely produced superior results compared with the Mathematizing Mode.
3. In the area of student attitudes and reactions towards the Mathematizing Mode, the methods previously used by the teachers were slightly more popular than the Mathematizing Mode.
4. Teacher reactions to the Mathematizing Mode were very favourable with the reservation that this method was perhaps too time-consuming.

In summary, it seems that the Expository Mode was definitely a superior treatment compared with the Mathematizing Mode. In the area of motivation to learn mathematics, the Mathematizing Mode showed promise for some students. However, it is likely that the objectives for teaching mathematics, cited earlier (page 13), were achieved better through the Expository Mode rather than the Mathematizing Mode.

Discussion

Without wishing to make excuses for the inferiority of the Mathematizing Mode with respect to student achievement, there are a number of factors which may have contributed to its inferiority.

1. There is the theory of Ausubel which states, in effect, that discovery learning is of little value to the student of high school age. The Expository Mode may, in fact, be the more effective for promoting student mathematics achievement in the cognitive areas of Bloom's Taxonomy.
2. The Mathematizing Mode was, in spite of a "warm-up" period, essentially unfamiliar to the students; that is, it was the method most different from their previous experience.
3. Teacher unfamiliarity with the Mathematizing Mode may have been a factor. While it is known that teacher behaviors were different in a Mathematizing class compared to an Expository class, it is not known whether or not teachers performed as effectively as possible with the Mathematizing Mode. There seems little doubt that teachers were quite familiar with the Expository Mode.
4. A factor related to teacher unfamiliarity with the Mathe-

matizing Mode is the possibility that the lack of evaluation of ideas as formed may have caused "misdiscoveries" which ought to have been "unlearned" before correct responses on achievement tests were forthcoming.

5. The duration of the experiment may also have been a factor. Was seven weeks long enough or too long to properly reveal and derive the benefits of a discovery method? The optimum length of time for a discovery approach is yet to be determined.
6. Another factor may have been the influence of an external examination in mathematics which faces students in the Edmonton Public School System at the conclusion of high school. Many students are very concerned about their results on this coming examination, and thus, some students in the sample population may have found the discovery approach time-consuming and anxiety-causing. This, in turn, may have influenced their performance on achievement tests.
7. The achievement tests themselves may have contributed to the inferiority of the Mathematizing Mode in this study. As indicated earlier (page 38), the post-test and retention test mainly sampled the Application, Analysis and Synthesis categories of Bloom's Taxonomy. These areas involve problem-solving skills which may be more difficult to influence and modify than skills related to Bloom's Knowledge and Comprehension categories. Seven weeks duration of exposure to the Mathematizing Mode may not

have been sufficient to observe any improvement in the Application, Analysis and Synthesis areas.

Although all teachers were involved in the same in-service sessions acquainting them with the teaching modes, they may have interpreted the treatments differently. Thus, the various interpretations by the teachers of the Mathematizing Mode and the Expository Mode may have influenced the results.

The results of the experiment are limited to the student population (college-preparatory pupils) and teachers involved in this particular instance. Since teachers and students were not randomly selected from a larger population, the results of this study are limited to a population for which the sample of teachers and students is representative.

A further restriction on the results of this study is the extent to which the content used in this instance (one small segment of the high school college-preparatory mathematics curriculum) was representative of the total mathematics curriculum.

Implications for Edmonton Public Schools

On the basis of the evidence in this study, there is little rationale for public high school teachers employing a discovery approach similar to the Mathematizing Mode in the teaching of quadratic functions and equations to the typical Mathematics 20 class, with the objective of improving student achievement. This study seems to indicate that a well-thought-out expository approach is more effective in promoting student achievement on reasonably typical teacher-constructed tests than is a discovery approach.

Recommendations for Further Research

Much research is needed in the area of discovery versus expository teaching in mathematics. Researchers can select from a great number of problems to form experiments of interest and value. The following questions and suggestions are a few possibilities.

1. Studies similar to this one, conducted at all high school grade levels with students of various ability levels.
2. The length of duration of the experiment should be varied. Student achievement as the result of the Mathematizing and Expository approaches could be compared after three, six or nine months of experimental treatments.
3. The objectives of mathematics instruction could be analyzed theoretically to determine which are possibly best achieved through use of the Mathematizing Mode alone, a "mixture" of the Mathematizing and Expository Modes, or the Expository Mode alone. This could be tested by a variety of studies similar in many respects to the present one.
4. Under the category of "when to use the Mathematizing Mode and when to use the Expository Mode", many possibilities exist. Perhaps the mainstream of the mathematics curriculum is best taught by use of the Expository Mode, while some areas, such as problem-solving, are best taught by use of the Mathematizing Mode. Perhaps a better mathematics curriculum can be established by determining what portion of a given curriculum should be taught using the Mathematizing Mode.

5. The area of teacher preparation for effective deployment of the Mathematizing Mode offers many opportunities. What characteristics and classroom behaviors mark the effective Mathematizing teacher? An answer to this question alone would make research similar to the present study much more valuable.
6. A critical area of discovery versus expository experiments in mathematics education is the evaluation of student achievement. What sort of examinations can be developed to evaluate student achievement without bias towards any teaching method? The present study utilized tests sampling several of Bloom's categories. Perhaps in the future, researchers will further explore this possibility, utilizing examinations made up of sections which definitely reveal a student's capabilities according to the various levels or categories of educational objectives.

Final Statement

This study would indicate a superiority for the defined Expository method over the defined Mathematizing method. Many other studies varying parameters both in the independent variables and dependent variables are yet needed to establish the place and value of "discovery" techniques in secondary school mathematics education.

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APPENDIX

APPENDIX A

DESCRIPTION OF EXPOSITORY MODE

EXPOSITORY MODE: Some ideas by Professor T. Kieren

Basic Philosophy

1. There are definite answers to things and best methods or rules for bringing these out.
2. The teacher acts as the agent with the answers.
3. The teacher exposes

   student looks, thinks, acts

   teacher evaluates.
4. In general, lessons are neat packages never going over one period.

Presentation

1. Basically a lecture or structured discussion accompanied by:
   1. Visual materials
   2. easily noted important points or rules
2. If you wish to develop and expose pattern notions
   1. you choose the pattern
   2. kids just fill it in
   3. teacher gives a polished form of rule although there may be controlled discussion.
3. Developmental guides:
   1. always state the purpose of the lesson in detail -- even give key ideas.
   2. use some organizing principle or ideas.

3. develop lesson to answer student questions or problems.
4. structure any student work for them.
5. always aim to have teacher stated rules or conclusions.
6. always give homework which practice and apply rules or conclusions -- always answer homework -- never give open-ended assignments.

Interaction

1. accept questions
2. answer questions
3. or refer to class for an evaluated (by you) answer
4. never go off on tangents; deliberately defer and redirect
5. ask answerable questions and try to get clear, precise answers
6. evaluate student responses or aim for complete response
7. if you ask for a wide open question ask it rhetorically: follow by a planned discussion which organizes around or answers your question
8. always highlight or note especially good responses
9. always summarize student responses with a rule or method which you label as best; try to encourage the use of this "best" method.

(p.s. - If you have other comments please make a note of them for discussion.)

APPENDIX B

TEACHER BEHAVIOR DURING TREATMENTS

A fellow-investigator, William Naciuk, had as a segment of his part of the project, the task of determining whether the teachers' behavior in a Mathematizing class was actually different from their behavior in an Expository class. To get at this task, Naciuk developed two instruments, an Observer Rating Scale and a Student Inventory.

> The Observer Rating Scale was used by the investigator to obtain measures of teaching methods during two live observations of each teacher's lessons in each treatment class before the in-service, during the experimental period, and after the experiment. The Student Inventory, which was designed to parallel the first instrument, was administered to all students during each of the above periods to elicit student opinions on the nature of the classroom activities. The pre-, during-, and post-scores so obtained from each instrument were translated into ranks. Rank orders were analyzed for differences Between Teachers and Within Teachers for methods. (Naciuk, 1968, Abstract)

Using these instruments, Naciuk found that the teacher behaviors were indeed significantly different in their two classes. (See Tables XIV and XV following)

TABLE XIV

WILCOXON MATCHED-PAIRS SIGNED-RANKS TEST ON DURING EXPERIMENT CATEGORY MEANS FOR TREATMENT CLASSES BY TEACHERS FROM OBSERVER RATING SCALE

Category	Teacher													
	1		2		3		4		5		6		7	
	Math	Exp	Math	Exp	Math	Exp	Math	Exp	Math	Exp	Math	Exp	Math	Exp
A	55	25	57.5	27.5	62.5	25	40	35	60	30	57.5	30	57.5	27.5
B	67.5	32.5	64	35	62.5	35	47.5	35	72.5	35	62.5	35	62.5	37.5
C	55	40	62.5	42.5	57.5	42.5	50	52.5	62.5	42.5	55	42.5	62.5	50
D	67.5	37.5	75	37.5	80	40	52.5	45	72.5	37.5	67.5	42.5	72.5	50
E	70	42.5	62.5	42.5	80	50	55	55	75	50	67.5	52.5	72.5	55
F	57.5	30	50	27.5	52.5	27.5	40	27.5	60	30	45	35	47.5	27.5
Wilcoxon T	0		0		0		1		0		0		0	
Prob.	$p < .01$		$p < .01$		$p < .01$		$.05 < p < .10$		$p < .01$		$p < .01$		$p < .01$	

(Naciuk, 1968, p. 92)

TABLE XV

WILCOXON MATCHED-PAIRS SIGNED-RANKS TEST ON TEACHERS' DURING-EXPERIMENT SCORES FROM STUDENT INVENTORY

Category	Teacher													
	1		2		3		4		5		6		7	
	Math	Exp	Math	Exp	Math	Exp	Math	Exp	Math	Exp	Math	Exp	Math	Exp
A	54	28	68	20	62	24	30	37	56	31	64	23	57	18
B	63	27	67	26	60	31	39	34	66	38	66	35	62	28
C	60	49	64	44	59	53	54	54	63	48	57	39	62	44
D	62	39	80	35	81	42	54	50	68	44	74	42	70	39
E	71	45	79	55	82	63	59	59	85	75	77	62	74	57
F	52	31	59	20	49	26	31	28	55	34	54	30	49	23
Wilcoxon T	0		0		0		4		0		0		0	
Prob.	$p < .05$		$p < .05$		$p < .05$		$p > .20$		$p < .05$		$p < .05$		$p < .05$	

(Naciuk, 1968, p. 93)

> The scores from the two treatment methods were significantly different if the obtained Wilcoxon T = 0. Tables XIV and XV both show that the Category Index scores for all teachers but Teacher 4 yielded a Wilcoxon T = 0, as obtained from the Observer Rating Scale and the Student Inventory. (Naciuk, 1968, p. 91)

APPENDIX C

CONTENT TAUGHT

The subject matter content included in the project is best indicated by the following copy of the table of contents concerning the relevant chapters nine and ten from the Mathematics 20 text, "Secondary School Mathematics Grade Eleven", Edition 2, by Beesack, MacLean et al.

IX The Quadratic Function and its applications

- 9.1 The quadratic function
- 9.2 The general quadratic function
- 9.3 The role of a in the general quadratic function
- 9.4 Graphs of quadratic functions
- 9.5 The solution of the corresponding quadratic equation
- * 9.6 Quadratic equations with literal coefficients (supplementary)
- 9.7 Maximum or minimum values, range, and graphs of quadratic functions
- 9.8 Maximum and minimum value problems
- 9.9 The general quadratic function (supplementary)
- 9.10 The range of the general quadratic relation, the equation of the axis of symmetry, and the coordinates of the vertex of the graph (supplementary)
- 9.11 Discovery of the general quadratic formula
- 9.12 The general quadratic formula
- 9.13 Problems involving the solution of quadratic equations

9.14 The graphical interpretation of the roots of a quadratic equation

* 9.15 Introduction to complex numbers

9.16 Equations solved as quadratic equations

9.17 Equations involving radicals solved as quadratics

9.18 Graphs defined by quadratic inequalities

X Introduction to the Theory of Quadratic Equations

10.1 Character of the roots of a quadratic equation

10.2 The sum and product of the roots of the general quadratic equation

10.3 Formation of quadratic equations given the roots

10.4 The formation of quadratic equations whose roots are related to the roots of a given equation (supplementary)

10.5 The maximum number of roots of a given equation (supplementary)

10.6 The roots of polynomial equations (supplementary)

* indicates sections omitted by most teachers in the project.

APPENDIX D

TESTS USED

MATHEMATICS 20 REVIEW TEST

TIME: 40 minutes

This is a multiple-choice test covering general knowledge of Chapters I-IV in Mathematics 20. It will be a good indicator of what you know about these chapters.

At the top of the answer sheet, please fill in the required information, writing "Math. 20 Review Test" after Name of Test. Answer the questions as indicated on the Answer Sheet. Use only Part I of the answer sheet. Please do not mark the question sheets in any way. Work each question carefully using the scratch paper, then choose the best answer. There is no penalty for guessing, so don't leave any questions out! Good luck!

Below are three mathematical statements of laws of exponents. Match the correct verbal name on the right to the mathematical statement on the left.

1. $a^n \div a^m = a^{n-m}$
2. $(a \cdot b)^n = a^n b^n$
3. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(a) Power of a quotient
(b) Power of a product
(c) Law of a quotient
(d) Negative exponent

4. The value of $(5x)^o$ is

(a) 0
(b) 5
(c) 1
(d) none of these

5. If set A contains p elements and set B contains q elements, then the cross product A X B has

(a) $\frac{p}{q}$ elements (c) p + q elements

(b) q^2 elements (d) pq elements

6. Which of the following sets of ordered pairs selected from (1,2), (2,6), (18,3), (0,0), (4,32) belong entirely to the relation $Q = \left\{(x,y) \mid 2x^2=y,\ x,y \in N\right\}$?

(a) (1,2), (0,0), (4,32) (c) (1,2), (18,3), (4,32)

(b) (18,3), (2,6), (1,2) (d) (2,6), (0,0), (1,2)

7. "If $a > b$ and $b > c$, then $a > c$" is known as the

(a) completeness property (c) trichotomy property

(b) transitive property (d) associative property

8. Assuming that you are to solve for x in each case, an example of an equation with literal coefficients is:

(a) $3(x-2)(x+4) - (3x+4)(x+2) = 3$ (c) $a^2x - a = b^2x - b$

(b) $\frac{1}{2}x^2 + \frac{3}{4}x^3 = 6$ (d) $\frac{x+1}{x-1} = \frac{5-2x}{7-2x}$

9. The value of $\sqrt{80}$ in simplest radical form is

(a) $4\sqrt{5}$ (c) $2\sqrt{20}$

(b) $5\sqrt{4}$ (d) none of these

10. The factors of $4x^4 - 9x^2 + 4$ are

(a) $(2x+1)(x-2)(2x^2+x+2)$ (c) $(2x^2-x-2)(2x^2+x-2)$

(b) $(4x^2-4)(x^2-1)$ (d) $(2x-1)(x+2)(2x^2+x+2)$

11. "If $a,b \in R$, then precisely one of the following alternatives holds: $a < b$, $a = b$ or $a > b$." This property is known as the

(a) trichotomy property (c) completeness property

(b) transitive property (d) associative property

12. The equation $\frac{x+1}{x-1} = \frac{5-2x}{7-2x}$, $x \in R$, has a solution under the conditions:

(a) $x=1$ and $x=3\frac{1}{2}$ (c) $x \neq -1$ and $x \neq 2\frac{1}{2}$

(b) $x=-1$ and $x=2\frac{1}{2}$ (d) $x \neq 1$ and $x \neq 3\frac{1}{2}$

13. The root of the equation $\frac{x+3}{3} = 1 + \frac{2x-4}{4}$, $x \in R$ is:

(a) 6 (b) -6 (c) $\frac{23}{2}$ (d) $-\frac{23}{2}$

14. For the relation $\left\{(x,y) \mid y \geq 2x+3,\ x,y \in R\right\}$ the x-intercepts are:

(a) $\left\{x \mid x \in R\right\}$ (c) $\left\{y \mid y \in R\right\}$

(b) $\left\{y \mid y \geq 3, y \in R\right\}$ (d) $\left\{x \mid x \leq -\frac{3}{2},\ x \in R\right\}$

15. An algebraic sentence in x and y which defines the function $A = \left\{(-2,-1),\ (-1,0),\ (0,1),\ (1,2)\right\}$ is:

(a) $y=x+1,\ -2 \leq x \leq 1, x,y \in R$ (c) $y=x+1,\ -2 \geq x \geq 1, x,y \in R$

(b) $y=x+1,\ -2 \geq x \geq 1, x,y \in I$ (d) $y=x+1,\ -2 \leq x \leq 1, x,y \in I$

16. The function, $g: x \rightarrow 5-x$, $x \in R$ is

(a) many-to-one (c) constant

(b) one-to-one (d) none of these

17. The factors of a^3-27b^6 are

(a) $(a^2-3b^3)(a+9b^2)$

(b) $(a^2+3ab^2+9b^4)(a-3b^2)$

(c) $(a-3b^2)(a^2-6ab^3+9b^4)$

(d) $(a-3b^2)(a^2-3ab^2+9b^4)$

18. The results of dividing $(6x^{3m+3}-3x^{2m+2})$ by $3x^{m+1}$ are

(a) $2x^{5m+5} - 1^{3m+3}$

(b) $2x^{2m+2} - x^{m+1}$

(c) $2^{2m+2} - 3x^{2m+2}$

(d) $6x^{3m+3} - x^{m+1}$

19. For the linear relation $L = \left\{(x,y) \mid y-x=3,\ x,y \in R\right\}$ the range is

(a) $y = 3$

(b) $\left\{x \mid x \in R\right\}$

(c) $\left\{y \mid y \in R\right\}$

(d) none of these.

20. The graph of the relation $\left\{(x,y) \mid y=x^2, x,y \in R\right\}$ is symmetric about

(a) neither axis

(b) the x-axis

(c) none of these

(d) the y-axis

21. For the function H defined by $H(x) = \frac{x+3}{x-2}$, $x \neq 2$, $x \in R$, compute $H(\frac{1}{5})$.

(a) $\frac{9}{16}$

(b) $\frac{15}{11}$

(c) $-\frac{16}{9}$

(d) $\frac{9}{11}$

22. If $f = \left\{(x,y) \mid y=4x,\ 0 \leqslant x \leqslant 4,\ x \in R\right\}$, then the range of f^{-1} is:

(a) $\left\{y \mid 0 \leqslant y \leqslant 16, y \in R\right\}$

(b) $\left\{y \mid 0 \leqslant y \leqslant 4, y \in R\right\}$

(c) $\left\{y \mid 4 \leqslant y \leqslant 16, y \in R\right\}$

(d) $\left\{y \mid 0 \geqslant y \geqslant 4, y \in R\right\}$

23. The domain of the relation $\left\{(u,v) \mid u^2+v^2=25, u,v \in R\right\}$ is:

(a) $u = \pm 5$

(b) $\left\{u \mid -5 \leqslant u \leqslant 5, u \in R\right\}$

(c) $u = \pm\sqrt{25-v^2}$

(d) $v = \pm 5$

24. One factor of $x^3 - 7xy^2 + 6y^3$ is

(a) $x - y$ (c) $x + 2y$

(b) $x + y$ (d) $x - 3y$

25. $a^{3m} \cdot a^{-m} + (a^{m} \div a^{-m})$ expressed with positive exponents is:

(a) a^{4m} (c) $\frac{1}{a^{3m^2}} + a^{2m}$

(b) $2a^{2m}$ (d) $2a^{4m}$

26. $\frac{2}{6-\sqrt{5}}$ expressed in simplest form with rational denominator is

(a) $\frac{12+2\sqrt{5}}{11}$ (c) $\frac{12+2\sqrt{5}}{61}$

(b) $\frac{12-2\sqrt{5}}{31}$ (d) $\frac{12+2\sqrt{5}}{31}$

27. Solve the following equation for a, expressing each root with rational denominator: $a\sqrt{2} = \sqrt{3}$.

(a) $a = \frac{\sqrt{3}}{\sqrt{2}}$ (c) $a = \frac{\sqrt{2}}{\sqrt{3}}$

(b) $a = \frac{\sqrt{5}}{2}$ (d) $a = \frac{\sqrt{6}}{2}$

28. Below is a logical argument which leads to an incorrect conclusion, as a result of performing an undefined operation. At which step, (a), (b), (c), or (d) does the undefined operation occur? (Hint: Remember $x = y$ at each step.)

Let $x = y$, $x \neq 0$ and $y \neq)$

Multiply by x: $x^2 = xy$

(a) Subtract y^2 $\qquad x^2 - y^2 = xy - y^2$

(b) Factor $\qquad (x+y)(x-y) = y(x-y)$

(c) Divide by $(x-y)$ $\qquad x + y = y$

(d) Substitute $x = y$ $\qquad y + y = y$

or $\qquad 2y = y$

Divide by y $\qquad 2 = 1$

29. If $f(x) = \frac{x-1}{x^2+1}$, find $\frac{f(0)+f(1)}{f(2)}$

(a) $\frac{1}{5}$ $\qquad$ (c) 0

(b) -5 $\qquad$ (d) $-\frac{1}{5}$

MATHEMATICS 20 TEST ON QUADRATICS

TIME: 40 MINUTES

This is a multiple-choice test covering general knowledge of quadratics in Mathematics 20.

At the top of the answer sheet, please fill in the required information, writing "Math. 20 Quadratics Test" after Name of Test. Answer the questions as indicated on the Answer Sheet. Use only Part I of the answer sheet. Please do not mark the question sheets in any way. Work each question carefully using the scratch paper, then choose the best answer. There is no penalty for guessing, so don't leave any question out! Good luck!

* * * * * * * * * * * * *

1. The equation of the axis of symmetry of the graph of $\left\{(x,y) \middle| y = -2x^2 - 3x + 14,\ x \in R\right\}$ is:

(a) $x = \frac{3}{2}$ (c) $x = -\frac{3}{4}$

(b) $x = -\frac{3}{2}$ (d) $x = \frac{3}{4}$

2. The maximum value of $f(x) = -(x - 3)^2 + 11$ is:

(a) 3 (c) 11

(b) -11 (d) -3

3. The solution of $2x^2 - 32 = 0$ is:

(a) $\{4\}$ (c) $\{4, -4\}$

(b) $\{-8\}$ (d) $\{0, 16\}$

4. If one root is 2, the missing term in $x^2 + 10x$ - _____ = 0 is:

(a) -24 (c) 12

(b) 24 (d) -12

5. In order that the graph of the function $x^2 + px + 25$ may be tangent to the axis of x, the value of p must be

(a) ± 5

(c) ± 10

(b) none of these

(d) 0

6. The graph of $ax^2 + bx + c$ is symmetrical about the y-axis when

(a) $c = 0$

(c) $b = 0$

(b) $a = b$

(d) a and b have the same sign

7. To complete the square in $t^2 - 7t = 3$, you should add to both sides:

(a) 49

(c) 9

(b) $\frac{49}{4}$

(d) $\frac{49}{2}$

8. The solution set of $x^2 - 5x + 5 = 0$ is

(a) $\left\{0, 5\right\}$

(c) $\left\{\frac{5 \pm \sqrt{30}}{2}\right\}$

(b) $\left\{\frac{5 \pm \sqrt{5}}{2}\right\}$

(d) none of these

9. The blank in the equation $x^2 +$ ______ $x - 14 = 0$, given that one root is -4, is best filled by

(a) $\frac{7}{2}$

(c) $\frac{1}{2}$

(b) $-\frac{7}{2}$

(d) $-\frac{1}{2}$

10. If $y = ax^2 = 6x + c$ has a minimum value when $x = 3$, a must equal

(a) 1

(c) 3

(b) 2

(d) $\frac{1}{4}$

11. The completed square form of $y = \frac{1}{3}x^2 + 2x - 5$ is

(a) $\frac{1}{3}(x + 3)^2 - 8$

(b) $\frac{1}{3}(x + 3)^2 + 8$

(c) $\frac{1}{3}(x - 3)^2 - 8$

(d) $\frac{1}{3}(x - 3)^2 + 8$

12. The graph of the function $P = \left\{(x,y) \middle| y = 3x^2 + 5x - 2,\ x,y \in R\right\}$

(a) crosses the x-axis twice

(b) is entirely about the x-axis

(c) is a tangent to the x-axis

(d) is entirely below the x-axis

13. The roots of the equation: $x^2 - 8x + 9 = 0$ are:

(a) real, unequal, irrational

(b) unreal, unequal, irrational

(c) real, equal, irrational

(d) real, equal, rational

14. For any non-zero real numbers a, b, and c, which of the following has a graph that goes through (0,0)?

(a) $ax^2 + bx + c = y$

(b) $ax^2 + c = y$

(c) $x^2 + bx + c = y$

(d) $ax^2 + bx = y$

15. If one root is 8, the missing coefficient in $x^2 - \_\_\_\_ x - 24 = 0$ is

(a) -5

(b) 3

(c) 5

(d) -3

16. Given that $6x^2 - 5x + k = 0$ has real roots, the values of k are:

(a) $k \leqslant \frac{25}{24}$

(b) $k \geqslant \frac{25}{24}$

(c) $k > \frac{25}{24}$

(d) $k < \frac{25}{24}$

17. Solve for x: $3x^2 - 5x - 11 = 1$

(a) $x = \frac{4}{3}$ and -3 (c) $x = -1$ and 4

(b) $x = -\frac{4}{3}$ and 3 (d) $x = 1$ and -4

18. The equation whose roots are $7+\sqrt{11}$ and $7-\sqrt{11}$ is

(a) $x^2 + 14x - 60 = 0$ (c) $x^2 - 14x - 38 = 0$

(b) $x^2 - 14x + 38 = 0$ (d) $x^2 - 14x + 2\sqrt{11}\,x + 38 = 0$

Given the function $y = x^2 - 4x - 5$, in the range $-3 < x < 7$, $x \in R$

19. The x-intercepts of the graph of the function are:

(a) -1 and 5 (c) -5 and 1

(b) -2 and 3 (d) 3 and -2

20. The coordinates of the vertex of the graph of the function are:

(a) $(0,-5)$ (c) $(2,-9)$

(b) $(2,7)$ (d) $(-2,9)$

21. For what values of x is y positive?

(a) $x > -1$ and $x > 5$ (c) $x < -1$ and $x < 5$

(b) $x < -1$ and $x > 5$ (d) $x > -1$ and $x < 5$

22. For what values of x is y decreasing as x increases?

(a) $x > -1$ and $x > 5$ (c) $x > 2$ and $x < -2$

(b) $x < -1$ and $x > 5$ (d) $-3 < x < 2$

23. Solve for x: $x^2 - 4x - 3 = -6$ (note the graph of the above function is very useful in this case)

(a) $x = 1$ and $x = 3$ (c) $x = 2 \pm \sqrt{7}$

(b) $x = -1$ and $x = -3$ (d) none of these

24. Find two positive real numbers whose sum is 13, if the sum of their squares is a minimum.

(a) The numbers are 6 and 7. (c) The numbers are $\frac{13}{2}$ and $\frac{13}{2}$.

(b) The numbers are $\frac{15}{2}$ and $\frac{11}{2}$. (d) The numbers are 0 and 13.

25. The values of r for which $rx^2 - 4x + 3 = 0$ has real roots are

(a) $r \geq \frac{4}{3}$ (c) $r \leq \frac{4}{3}$

(b) $r < \frac{4}{3}$ (d) $r > \frac{4}{3}$

26. If $2x^2 - ax + 8 = 0$ has equal roots, the value of a is:

(a) ±8 (c) 4

(b) 64 (d) 16

27. The vertex of the graph of $y = 2x^2 + 8x + 5$ is:

(a) $(2,-5)$ (c) $(-2,5)$

(b) $(-2,3)$ (d) $(-2,-3)$

28. Given the function $y = -x^2 - 7x - 12$ the completed square form is:

(a) $-(x + \frac{7}{2})^2 - \frac{97}{4}$ (c) $-(x + \frac{7}{2})^2 + \frac{1}{4}$

(b) $-(x + \frac{7}{2})^2 + \frac{49}{4}$ (d) $-(x + \frac{7}{2})^2 + \frac{25}{2}$

29. The sum of the roots of an equation $ax^2 + bx + c = 0$ is 4. If $a = -1$, then $b = ?$:

(a) 4 (c) 1

(b) 3 (d) -3

30. The y-axis is the axis of symmetry of $y = 4x^2 - 3kx - 15x$.

The value of k must be

(a) $-\frac{7}{3}$ (c) $-\frac{15}{4}$

(b) -5 (d) 5

MATHEMATICS 20 TEST ON QUADRATICS (R)

TIME: 40 Minutes

This is a multiple-choice test covering general knowledge of quadratics in Mathematics 20.

At the top of the answer sheet, please fill in the required information, writing "Math. 20 Quadratics Test (R)" after Name of Test. Answer the questions as indicated on the answer sheet. Use only Part I of the answer sheet. Please do not mark the question sheets in any way. Work each question carefully using the scratch paper, then choose the best answer. There is no penalty for guessing, so don't leave any question out! Good luck!

* * * * * * * * * * * *

1. The solution set of $3x^2 - 27 = 0$ is:

   (a) $\{3\}$ (c) $\{3,-3\}$

   (b) $\{3,3\}$ (d) none of these

2. If one used the quadratic formula to solve $3x^2 - x - 4 = 0$, the numeral under the radical would be:

   (a) -49 (c) -47

   (b) ± 7 (d) 49

3. The roots of the equation $3x^2 - 8x + 9 = 0$ are

   (a) complex (c) real, equal, irrational

   (b) real, unequal, irrational (d) real, unequal, rational.

4. If one root is 4, the missing term in $x^2 - 7x +$ ____ $= 0$ is

   (a) 12 (c) -12

   (b) 3 (d) -3

5. The x-intercepts of the function: $y = x^2 + 3x - 4$ are

(a) 2 and -2

(b) 0 and 4

(c) 1 and -4

(d) -4 and -1

6. Given $y = x^2 + 3x - 4$; y is negative when:

(a) $-4 < x < 1$

(b) $1 < x < -4$

(c) $-1 < x < 2$

(d) $2 < x < -1$

7. One of the roots of $2t^2 - 5t + 2 = 0$ is

(a) $\frac{1}{2}$

(b) 4

(c) 5

(d) $-\frac{2}{3}$

8. The equation of the axis of symmetry of the graph of $y = 3x^2 - 4x + 14$ is:

(a) $x = \frac{2}{3}$

(b) $x = -\frac{2}{3}$

(c) $x = -\frac{4}{3}$

(d) $x = \frac{4}{3}$

9. The coordinates of the vertex of the graph of $y = x^2 - 4x + 3$ are:

(a) (4,3)

(b) none of these

(c) (-2,1)

(d) (2,-1)

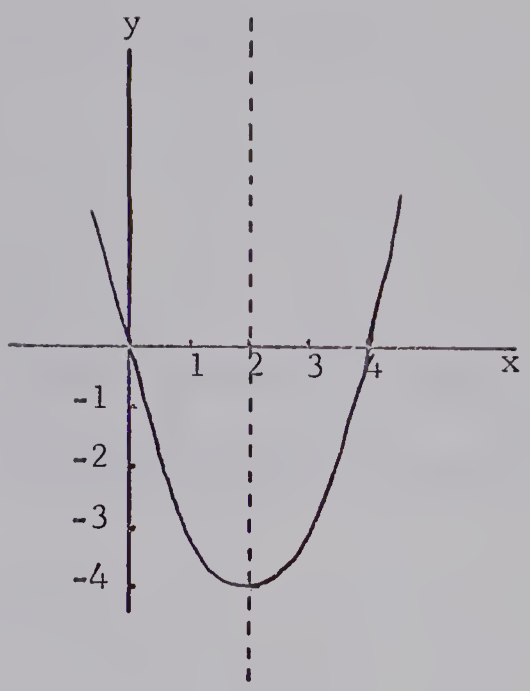
10. An equation whose roots are 4 and $\frac{1}{2}$ is

(a) $x^2 + \frac{9}{2}x + 2 = 0$

(b) $2x^2 - 9x + 4 = 0$

(c) $2x^2 + 9x + 4 = 0$

(d) $x^2 - \frac{9}{2}x - 2 = 0$



11. Here is the graph of $y = x^2 - 4x$.

Use it to solve $x^2 - 4x = -3$.

(a) $x = 0$ and 4 (c) $x = 1$ and 3

(b) $x =$ none of these (d) $x = 2$ and 0

12. The vertex of the graph of $y = x^2 + 2x - 8$ is:

(a) (1,-5) (c) (-1,-9)

(b) (-1,-5) (d) (-2,-8)

13. If the roots of $2x^2 - bx + 2 = 0$ are equal, the value of b is:

(a) 16 (c) ±2

(b) ±4 (d) -8

14. For any real numbers a and b, if a quadratic equation can be put into the form $(2x + a)(3x + b) = 0$, then one root of the equation is

(a) $\frac{b}{3}$ (c) a

(b) -b (d) $-\frac{a}{2}$

15. To complete the square in $x^2 + 4x = -2$, you should add to both sides of the equation:

(a) 3 (c) 4

(b) -4 (d) -2

16. If one root is 2, the missing term in $x^2 + 5x -$ _____ $= 0$ is:

(a) 7 (c) -14

(b) -7 (d) 14

17. The graph of the function $y = 2x^2 + 5x + 4$,

(a) crosses the x-axis twice (c) is entirely below the x-axis

(b) touches the x-axis only once (d) is entirely above the x-axis

18. The maximum value of $f(x) = -(x - 2)^2 + 3$ is:

(a) -3 (c) -2

(b) 3 (d) 2

19. If $8x^2 + kx + 2 = 0$ has real roots, what is the range of values of k?

(a) $|k| > 8$ (c) $|k| \leqslant 8$

(b) $|k| < 8$ (d) $|k| \geqslant 8$

20. Two points on a parabola are (6,-1) and (10,-1). Find the equation of the axis of symmetry for the parabola:

(a) $x = -1$ (c) $x = 6$

(b) none of these (d) $x = 7\frac{1}{2}$

21. An equation in the form $ax^2 + bx + c = 0$ whose roots are $\frac{1}{2}$ and $\frac{3}{4}$ is:

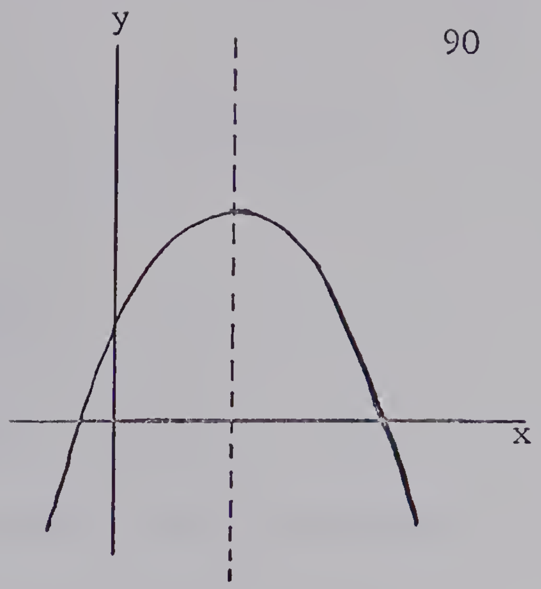
(a) $8x^2 - 10x + 3 = 0$ (c) $8x^2 - 8x + 3 = 0$

(b) $8x^2 + 10x - 3 = 0$ (d) none of these

22. From the sketch of the graph of $y = ax^2 + bx + c$ to the right, we know that

(a) $a < 0,\ c < 0$ (c) $c < 0,\ D = 0$

(b) $a < 0,\ b > 0$ (d) $a < 0,\ b < 0$



23. The completed square form of $y = -2x^2 + 4x - 3$ is

(a) $-2(x + 1)^2 + 2$ (c) $-2(x - 1)^2 - 1$

(b) $-2(x + 1)^2 + 5$ (d) $-2(x - 1)^2 - 5$

24. The graph of $ax^2 + bx + 2$ is symmetrical about the y-axis when

(a) $a = 0$ (c) $a = -b$

(b) $b = 0$ (d) a and b have the same sign

25. The equation $0 = 2x^2 - 5x + c$ will have complex roots if:

(a) $c > \frac{25}{8}$ (c) $c \geq \frac{25}{8}$

(b) $c > \frac{25}{4}$ (d) $c < \frac{25}{4}$

26. In order that the graph of the function $y = px^2 + 6x + 13$ may just touch the x-axis in one place, the value of p must be

(a) $\frac{9}{13}$ (c) $\frac{3}{26}$

(b) $-\frac{9}{13}$ (d) $\frac{13}{9}$

27. Given the function $y = -(x + \frac{7}{2})^2 + \frac{1}{4}$, the range of the function is

(a) $\{y \mid y \leq -\frac{7}{2}$ (c) $\{y \mid y \leq \frac{25}{2}\}$

(b) $\{y \mid y \leq \frac{1}{4}\}$ (d) $\{y \mid y \geq \frac{7}{2}\}$

28. If m and n are the roots of $x^2 + 5x + 3 = 0$, the value of $\frac{1}{m} + \frac{1}{n}$ is:

(a) $\frac{3}{5}$ (c) $-\frac{5}{3}$

(b) 5 (d) $-\frac{1}{2}$

29. If $y = ax^2 + 6x + 2$ has a maximum value when $x = 3$ is the axis of symmetry, a must equal

(a) -2 (c) -3

(b) -1 (d) none of these

30. A ball is thrown vertically upward with an initial velocity of 100 feet per second. Neglecting air resistance, the distance d feet from the release point in time t seconds is given by $d = -16t^2 + 100\,t$. Determine the maximum height d attained by the ball.

(a) $78\frac{1}{8}$ feet (c) none of these

(b) $148\frac{1}{4}$ feet (d) $156\frac{1}{4}$ feet

APPENDIX E

REACTIONS OF STUDENTS TO THE MATHEMATIZING MODE

(The following is a report prepared by J. Vance presenting the results of his investigations into student reactions to the Mathematizing Mode.)

DISCOVERY TEACHING AND THE STUDENT

Introduction

Educators generally agree that the learning process in a classroom is affected by many variables including the attitudes of both the teacher and the learner, interest of the learner in the subject matter, emotional climate of the classroom, and the method of instruction. Research on various methods of teaching indicates that there is no one best method for all instructors nor for all students, but that the effectiveness of different methods varies with individual characteristics such as intelligence, motivation, background, self-concept, self-confidence, and emotional adjustment.[1]

As part of a group study on discovery teaching, the writer attempted to assess student acceptance of and preference for the experimental method by means of a specially designed questionnaire and personal interviews.

Method

The subjects consisted of the students from 14 Math 20 classes taught by seven teachers in four Edmonton Public high schools. Each teacher taught one class by the Mathematizing Mode (MM), a discovery

method and one class by the Expository Mode (EM), a more traditional method. The experiment lasted about seven weeks, during which time the section of the regular mathematics course dealing with the quadratic function was taught to all classes.

At the end of the first week of the experiment all participating students were given the Methods Preference Scale (MPS), an investigator-prepared instrument designed to assess preference for a discovery-type method as opposed to an expository approach. The internal consistence of the MPS was found to be .68, and a test-retest reliability coefficient was .72. At the conclusion of the experimental period the MM students again completed the MPS to which an item directly related to their experience with the MM had been added. Teachers were asked to identify students who had responded most favorably to the MM and students who became highly anxious or who otherwise reacted negatively to the method. These people were interviewed by the investigator.

Results

1. Table I shows the group mean scores obtained by classes of students on the MPS for each administration of the instrument.

TABLE I

Teacher Class	1	2	3	4	5	6	7	T
EM	3.1	3.2	4.0	4.0	2.7	3.9	4.5	3.58
MM (1 wk)	3.6	3.6	3.9	4.2	4.3	3.5	4.7	3.97
MM (7 wks)	4.4			4.5	4.8	3.4	6.4	4.64

The data suggests a trend towards higher scores on the MPS (greater preference for a discovery method) with increasing experience with the MM.

2. The proportions of students who gave the "high discovery" response to the ten items on the MPS is reported in Table II.

TABLE II

Proportions				Significance of Differences of Proportions	
Item	EM	MM (1 wk)	MM (7 wks)	EM-MM1 z-scores	EM-MM7
1	.26	.35	.44	1.76	3.34 *
2	.36	.37	.46		1.75
3	.32	.37	.42		1.73
4	.18	.29	.36	2.25 *	3.61 *
5	.28	.34	.33		
6	.59	.58	.49		-1.67
7	.40	.47	.50		1.66
8	.35	.35	.45		1.62
9	.48	.62	.71	2.48 *	4.02 *
10	.34	.29	.41		

* $p < .01$

Again the pattern of responses indicated a trend toward a greater proportion of students giving the high discovery answer with increasing experience under the MM. The z scores[2] reveal that several of these differences are significant. Note the reverse trend in item 6.

3. A nonparametric sign test[3] showed that there was a significant increase in the MPS scores of the MM students over the seven week period in which they were taught by the MM.

4. Table III shows the responses of five classes of MM students to Item 11 of the MPS.

TABLE III

Teacher Response	1	4	5	6	7	T	Percentage
A	5	4	5	1	5	20	38%
B	7	9	4	5	7	32	
C	4	6	2	5	2	19	14%
D	5	7	10	4	5	31	48%
E	6	4	7	14	4	35	
Total	27	30	28	29	23	137	

The table shows that 38% of the students responding indicated that they would like to continue studying mathematics by the MM, 48% of the students said they would prefer to be taught by the method the teacher had used before, and the remaining 14% of the students did not care either way. It is noted that only in the class taught by teacher 6 is there a significant difference between the proportion of students liking the MM and the proportion who dislike it.

REFERENCES

1. C.E. Buxton, College Teaching: A Psychologist's View (New York: Harcourt, Brace and Company, 1956) p. 342.

2. H.M. Walker and J. Lev, Statistical Inference (New York: Hall, Rhinehart and Winston, 1953) pp. 77-79.

3. G.A. Ferguson, Statistical Analysis in Psychology and Education (Second Edition) (New York: McGraw-Hill Book Company, 1962) pp. 356-357.

Name________________________________

The purpose of this questionnaire is to see what students think about mathematics and how it could be most effectively learned. There are no right or wrong answers. In some questions you may agree or disagree with both coices, but in each case choose the one answer that seems best to you.

_______ 1. I think that math is a subject

A. that is "cut and dried."
B. in which I can express my own ideas and have them discussed.

_______ 2. I think that a math teacher's job is mainly

A. to help students discover principles and methods for themselves.
B. to explain principles and give methods for solving problems.

_______ 3. The best way to learn math

A. is by doing many type problems.
B. is by developing an idea or method yourself.

_______ 4. In math class I like

A. to make hypotheses and test them out.
B. to apply correct ideas and formulae given by the teacher.

_______ 5. During math class I feel I learn more

A. when I listen carefully and try to understand what the teacher is doing.
B. when I answer questions and offer suggestions.

_______ 6. I am most bored in math class

A. when the teacher tells me everything I have to know.
B. when I am supposed to come up with ideas on my own.

_______ 7. When I spend time trying to do a problem by using an idea I later discover to be incorrect, I feel I have

A. wasted my time.
B. still benefited mathematically.

______ 8. When we begin a new topic in math, I think that

A. there may be aspects of the topic that the teacher has never considered which the class can investigate.
B. the teacher should know all about what we will study.

______ 9. In the investigation of mathematical principles or problems

A. guessing plays a very important and useful role.
B. there is very little place for guessing.

______ 10. In math class I get most upset when

A. the teacher suggests that I find out for myself the answer to a question I have asked.
B. the teacher points out a mistake I have made before giving me a chance to find it myself.

______ 11. If I had my choice between continuing to study Math 20 as we have the past two months or having it taught as it was before, I would

A. strongly prefer to continue.
B. prefer to continue.
C. not care either way.
D. prefer the previous method.
E. strongly prefer the previous method.

The students who were interviewed were those who had been identified by their teachers as being either very satisfied and successful in using the Mathematizing Method (MM) or very dissatisfied with this method of studying mathematics in the classroom. Comments which characterized these two positions were: "It's been a marvellous experience. I'm glad I was in an experimental class." and "I hated every minute; I don't see how anyone could like it."

When asked to briefly describe the new method, the students usually gave one of the following answers: "The students figure out things for themselves instead of having the teacher tell them; you teach yourself; you discover rules and methods; the teacher did what we wanted her to instead of the other way around; the class makes the decisions, not the teacher."

Asked what they thought the purpose of the method was, students replied it was "to make you think; to get you to use your brain; to get everyone to participate; so you will understand and not just memorize; to enable you to remember better; to allow you to follow your own line of thinking."

The students were asked to point out some of the difficulties, disadvantages, and problems associated with the experimental method. Many students felt that it was difficult for them to adjust to this method in grade 11, that it was too much of a change from the way they had always been taught, and that it takes time to get used to it and to be able to use it effectively. They suggested that it be introduced in earlier grades or that a period of transition be allowed. As one fellow put it, "students are not used to thinking". Some classes apparently felt some apprehension about being involved in an

experiment and not learning the required material to pass the course.

Most of those questioned thought the method takes too long, and that it would be much easier and quicker for the teacher to simply tell the students what they had to know. Many students said they became frustrated when the teacher would not tell them an answer or indicate to them if they were right or wrong. They felt that this caused them to "learn incorrect methods, get confused or lost, and waste time." One girl said it was "maddening".

Some students complained that a few people did all of the participating and monopolized the discussions. A few students said that they couldn't follow the discussions and got behind. On the other hand several pupils said they were often bored because the teacher would dwell on a point they had already understood. Another complaint was that it is more difficult to learn from other students than from the teacher because students express their ideas poorly.

It is interesting to note that marks seemed to have little to do with whether a student liked or disliked the Mathematizing Mode. Many of the students who liked the method did so in spite of reduced grades, and several students who were opposed to this approach said that they had maintained their average in mathematics. In one class the boy most opposed to the method made the highest mark on the posttest.

Another interesting point is that, in general, those students who were unhappy with the MM said they believed it was a good way, perhaps the right way, to learn mathematics. They felt that it had not been good for them though. The following comments illustrate this view:

"I used to do well in math. This year I haven't been working too hard and the new method gave me the opportunity to sluff off even more. I sat back and let the others do the thinking, and only made 50% on the test."

"It is a better method -- but not for me. Give me a rule and lots of examples."

"I spent a lot of time at first but got discouraged. It didn't work for me."

"I was afraid I would give a wrong suggestion and have the class think I was stupid. So I just sat back and listened."

"I didn't want to get that involved in math. I just want to pass it."

"I was always in the dark -- I like to know where I'm going."

"It is good for the 'way-out type' with crazy ideas."

"You have to concentrate to benefit. This is hard to do every day -- especially right after dinner."

Students who liked the MM did so for several different reasons. Some of these are suggested in the following remarks:

"I felt better about math even though my mark went down. I used to be bored but now I'm really interested in math. I looked forward to it, did hours of homework. Even my parents noticed the change."

"I've always made good marks in math but this time I really understood it instead of just memorizing rules. The exam confused me though. I wanted to do the problems my ways but I couldn't decide which method to use."

"I repeated Math 10 and am now repeating Math 20. I used to

hate math. Now I like it as I can see the purpose. I made 96% in the last exam."

"I enjoyed showing my methods to the class and trying to find mistakes in others' methods. I felt I was discovering -- like Archimedes."

"_____________ used to be a poor math student, never do his homework. Now he works hard and gets good marks. He always comes up with crazy ideas -- but they work."

Several students said they think of math differently now. It is a subject which can be explored and discussed in class. Many of the pupils said they did not think they learned more math, but they had learned how to think, to participate, and to explore.

One student expressed surprise that he was able to learn by this method. He said he first used to think: "How can I be learning any math when I'm figuring everything out for myself? I thought only teachers knew the answers."

There was no agreement on the question "What type of student benefits most from this approach and is able to use it most effectively and enjoy it?" Although many students replied that it was the serious student or the honors student, others felt that the method gave a new and different group the opportunity to shine. The learner who had previously done well by memorizing was no longer outstanding, but "the individual who thinks and who likes to express ideas, the 'inventor-type', and the 'way-out type' now became a class leader." Some students replied that the method gave the weaker students a chance to catch up to the high achievers in mathematics, but others felt that students whose background in mathematics was weak were at an

even greater disadvantage. Most of those interviewed agreed that learning by this method required work, participation, and perseverence, and that the lazy or unaggressive student is left out and falls behind. Perhaps the most insightful answer to this question came from the boy who said that the experimental method works well for the student who is really interested in learning and understanding mathematics, but that it is a burden for those who merely want to pass the course to complete a requirement.

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